2021

Royal Society Mathematical Future Programme: Call for Views responses

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Executive summary:

The Royal Society's Mathematical Futures programme aims to identify the mathematical competences, defined broadly, needed by students leaving compulsory education and training in the future. It will consider how mathematics education for 4–19-year olds should consequently be reshaped, together with the implications of that, including the skills required for teachers. As a first step, a Call for Views ran October 2020-January 2021. This report is of the findings from that Call for Views. Data comprised seven stand-alone contributions arising from free-form responses or interview transcriptions, together with 191 questionnaire responses largely from across the mathematics, computing, substantial mathematics-user and mathematics education communities, and workshop responses from three cross-stakeholder validating webinars (nine focus groups) held in April 2021. *While there was small-scale but sometimes deeply reflective variation of response in some areas, as reported later, key themes reported that received wide support were:*

1. ***Individual and societal mathematical needs are already not being met***
2. ***Mathematics, and mathematical needs, are evolving. Changing roles for mathematics and the tools available have transformed its potential impact and the scale of need.***
3. ***Mathematics education should therefore be expansive and inclusive, but probably separate from computing education***
4. ***A variety of mathematical pathways would be needed to achieve this***
5. ***Our mathematics education workforce is thought to be significantly under-equipped and there are other attitudinal and systemic challenges to such a vision***
6. ***Addressing these challenges would require unprecedented collaboration and investment***
7. ***Critically, curricula, pedagogies and assessment would need coherent re-envisioning***

In more detail:

1. *Individual and societal mathematical needs are already not being met*

- There is compelling evidence that limitations in UK mathematical skillsets at all levels are already limiting economic and societal thriving, with activity in related fields often maintained by overseas-educated workers. This situation may not be sustainable and demand is likely to increase.
- Individual and societal thriving increasingly depends on all experiencing, and mastering at an appropriate level, mathematics as a creative, fallible, sometimes-demanding but often empowering route to interacting with the world around us.

2. *Mathematics, and mathematical needs, are evolving. Changing roles for mathematics and the tools available have transformed its potential impact and the scale of need.*

- Respondents reported a range of mathematical competences important to them in their work. They commonly identified deep and flexible numeracy, statistical understanding, problem solving and reasoning, modelling and increasingly, use of digital and data-related skills, together with some role-specific competences.
- There was wide agreement that mathematics has not fundamentally changed its nature in recent years – but that the roles it plays, and the tools available for mathematical activity, have increased considerably with the expansion of technological capacity and access, and explosion of easily-availability data.
- As a consequence, mathematical aspects of work can have more and wider impact, and their roles are more often in the public domain. Examples given were the developing use of algorithms for decision-

making, role of machine learning, and expansion of applications well beyond the sciences into social sciences, arts and humanities, including sometimes questionable use in the media.

- Mathematics has, however, continued to evolve, e.g. in category theory, and applications will continue to change as AI and technology expand further. As a consequence, mathematical work can have more and wider impact, and its roles are more often in the public domain.
- There is therefore a substantially enhanced need for mathematical equipping at all levels

3. *Mathematics education should therefore be expansive and inclusive, but probably separate from computing education*

- Mathematics education should equip all with basic mathematical, data (including statistical) and financial literacies, rooted in confident conceptual grasp and robust processes that include appreciation and experience of how technology can support those. Wider data science education might take place across the curriculum.
- Problem-solving, creative and critical thinking, reasoning and abstraction, underpinned by core knowledge, are central to mathematics. Young people need an appreciation of the power and importance of mathematics as integral to human culture and also the ability and inclination to use mathematics to empower their own lives. They should experience both satisfaction and enjoyment in their use of mathematics, often working collaboratively and cross-discipline, and develop productive dispositions towards its use, so that all come to identify as successful users of mathematics at an appropriate level.
- Digital skills, techniques and resources support and enrich use of mathematics: for example, confident use of appropriate software supports students in mathematical understanding of authentic data, and of risk. However, almost all thought computing per se and mathematics should be kept as separate but persistently complementary curriculum strands, especially as students mature. Computational and algorithmic thinking and AI, beyond immediate mathematical purposes, had a low profile in responses. Clarifying the language used around computing would support informed debate in related areas.

4. *A variety of mathematical pathways would be needed to achieve this*

- Young people's engagement with some form of mathematics education (defined broadly) until age 18, should be via flexible pathways and mathematical focus that together provide for the range of mathematical education needs identified.
- Such pathways should educate for at least 1. Mathematically functional citizens; 2. occupational users of mathematics; 3. specialist users of mathematics, including academic and professional mathematicians. Many need solid mathematical competences that go beyond basic mathematical, financial and data literacies, and some also need more advanced 'pure', 'applied' and 'applicable' mathematics.
- All should have opportunity to study to the level they will require for the future, in data-rich and digitally-enhanced ways, and to acquire further mathematics learning as individual and societal needs develop: some such additional learning could also be employment-based.
- There would be a challenge in devising structures that support timely, equitable and successful choices of mathematical pathways.

5. *Our mathematics education workforce is thought to be significantly under-equipped and there are other attitudinal and systemic challenges to such a vision*

- A central challenge of such an expansive vision for mathematics education is the recruitment, retention and continuing education of sufficient well qualified and prepared teachers of mathematics, as well as

the mathematical equipping of teachers of other subjects. (Compared with many developing and developed countries, our aspirations for mathematics teachers, particularly in England, are very modest).
- There is a need to further develop public perceptions so that mathematical competences are understood to be as achievable, and as central to individual and societal thriving, as literacy is, and digital technologies a natural tool in their use; such competences should be valued and rewarded.
- Other challenges include establishing a cross-party political will to prioritise, develop and sustain the changes needed, in ways deeply informed by collaboration across subject-specific and broader education system experts - and then maintaining an inclusive and stable curriculum that is supported by coherent pedagogies and assessments.

6. ***Addressing these challenges would require unprecedented collaboration and investment***
- There was frustration that a number of strongly-evidenced reports and policy initiatives focused on developing mathematics education, and teacher education, to meet even current mathematical and digital needs, have not borne significant fruit. Such shortcomings will have even greater impact on our individual and societal thriving if mathematical needs increase further, and it was thought impediments to full fruition need identification and addressing, if ambitious new visions are to be achieved.
- Re-shaping mathematics and digital education in the expansive ways envisaged would require unprecedented collaboration across the range of interested parties.
- It would need substantial, sustained investment in subject-specific and systematic teacher development that is career-long, and a cross-party political will to be seen to value mathematical competences.
- Only with such changes could we address the challenge of equitably equipping a diverse range of young people to choose to continue working with mathematics, at a variety of levels.

7. Critically, curricula, pedagogies and assessment would need coherent re-envisioning
- Mathematics curricula, pedagogies and assessment were thought to need radical and sustainable re-thinking in ways that are coherent with one another, and with the goals outlined. High quality resources also coherent with these would further support valid curriculum enactment. In this widely-embraced vision:
- Mathematics *curricula* would focus on mathematical concepts and key processes of mathematics: problem solving, reasoning and communication, as well as core skills, techniques and knowledge of elementary mathematics; they would address statistics and data literacies and contribute to developing digital literacies and computational thinking.
- Mathematics *pedagogies* would harness authentic examples, using digital technologies to focus on conceptual development and problem-solving; young people would be supported to develop rigorous reasoning and critical, collaborative and communication skills; pedagogies would be appropriate to the mathematical aspirations and maturity of learners, so that young people would both enjoy and appreciate mathematics as a cultural activity with satisfactions that transcend immediate needs.
- The main purpose of large-scale summative *assessments* would be to validate achievements, and their format in many cases would move away from timed written papers, towards more portfolio-based and other evidence that validated collaborative, problem-solving and often digitally-supported competences and possibly experiences.

1. Introduction

The Royal Society's work in education policy is focused on creating the conditions for young people to receive a broad and balanced education, which equips them with the knowledge, skills and resilience that will enable them to successfully navigate and thrive in an uncertain world that is being transformed by digital technologies expected to radically change the types of jobs that will be available in future.[1] The Society's policy work further addresses priority areas such as mathematical and quantitative skills, sciences and computing education. Whilst there has been a rapid increase in the demand for data science skills, there is uncertainty about when and which jobs will cease and what new types of jobs there will be. The mathematical needs of people in the UK should be better understood than they are now, and our education systems configured to provision them.

Accordingly, the Royal Society's Mathematical Futures Programme (MFP) Board has determined that the main aims of the MFP should be:

a) To analyse the mathematical competences that will be needed by students leaving compulsory education and training in the future;
b) To consider the implications of reshaping of mathematics education for 4–19 education as a whole and mathematics education in particular;
c) To recognise the skills required for teachers who would teach these curricula.

In the programme, the term **mathematics** is used inclusively, covering ways of thinking, reasoning and solving mathematical problems which touch on many aspects of everyday life, work and study (in mathematics as well as in other subjects/disciplines). It includes quantitative skills and other mathematical activities such as those associated with numeracy, statistics, computing, and data analysis. **Mathematical competences** cover the way mathematical concepts, skills and understanding as well as attitudes are brought together when applying mathematics to solve problems. The programme seeks to look at mathematical competences across disciplines, not only from different areas of natural science and mathematics, but also from the social sciences and humanities.

The MFP suggests that the mathematical needs of the population can be broken down into five overlapping categories of mathematical needs as follows:

1. Citizens requiring basic functional numeracy in their lives.
2. Citizens requiring the ability to consume and critique numerical claims.
3. Citizens in traditionally non-quantitative jobs/professions, but now requiring increased skills: e.g., lawyers, journalists, civil servants, politicians, health, administrators, teaching practitioners (not mathematics), vocational professions
4. Citizens in jobs/professions that require mathematical skills as a core component, e.g. analysts, engineers, financial professionals, scientists, social scientists.
5. Professional mathematical scientists, e.g. mathematicians, mathematics teachers.

A Call for Views questionnaire on the changing nature of mathematics and implications for education was launched on 9 October 2020, running until 11 January 2021. This was complemented by a range of four responses to interviews or workshops instigated by the Royal Society education team, and freeform responses from two key national bodies and two others. The aim of the Call for Views was stated to be the informing of

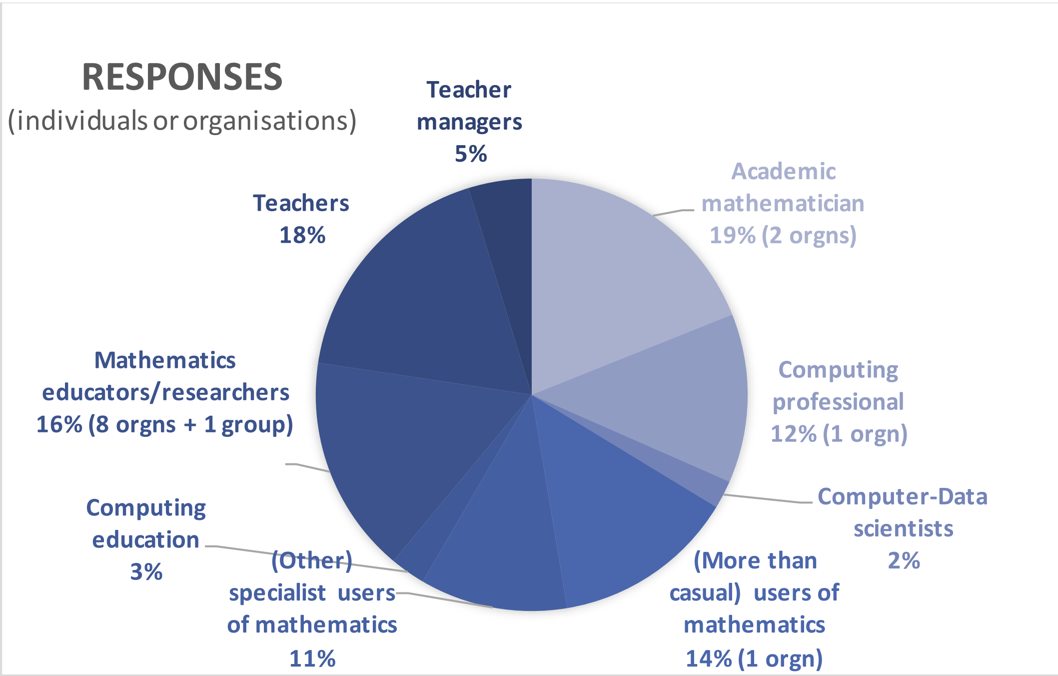
[1] https://royalsociety.org/news/2018/09/the-impact-of-ai-on-work/

subsequent stages of the project but it also served to publicise the initiative, and offered the range of immediate and other stakeholders, an opportunity to contribute their views.

This report presents an analysis of the totality of those views, initially structured by the questions asked, and moderated and complemented by input from a series of three 'stakeholder' webinars held in April 2021. We also contextualise views received at a high level, within recent national and international policy and debates about education, mathematics, research, and society, and by dominant relevant literature.

2. Approach

2.1 The online questionnaire resulted in 191 usable responses. These were systematically and iteratively organised into (inexact and porous) core categories of respondent, to highlight synergies in responses. Data synthesised for this report were based on questionnaire responses as follows:



2.2 Some of these categories cover a wide range of occupational demands and lenses, but any such analysis will inevitably involve compromises. Responses were variously from individuals, from groups of professionals, or from professional organisations, and where we quote, we distinguish between those. It was successful in attracting high quality and significant responses from a large proportion of professional organisations concerned with mathematics and mathematics education in the UK, as well as sustained and reflective responses from experts across stakeholder groups and employer users of mathematics. These data were supplemented by responses from a high-profile computing initiative (Wolfram), a computing professional association (the British Computing Society), and two home nation education departments (the Education Scotland response also drawing on views from mathematics advisers across Scotland), as well as transcriptions of interviews with an education charity and a national media outlet (the BBC), and notes from a workshop with two key funders. This report reflects input from the sum of those sources, as well as findings from the validating webinars referred to below.

2.3 The Mathematical Futures programme is targeted at the whole of the UK, but nearly all responses, even from UK-wide organisations, were focused on the English education system, and particularly on government power to change the curriculum and assessment model for England. There were two (informed and in-depth) group responses from Scotland, but none identifiably from Wales. One Northern Irish participant contributed to a webinar.

2.4 It is important to note that the sample achieved is opportunistic, and likely to be from those with an interest in the theme: awareness was promoted through a variety of mathematics-related professional and other organisations, but responses can in no way be interpreted as representative of specific parts of mathematics (education or user) communities, and the depths of responses varied considerably. This is a feature of a Call for Views, but the tool also offers the opportunity to acquire unstructured and rich responses informed by a range of expertise.

2.5 Initial open analysis and coding was within the above (emergent and iteratively reviewed) categories, with intra-category open analysis of data within research question, giving rise eventually to grounded within-, but often across-category themes. Finally, from those we were able to derive the seven overarching themes reported here. Emerging analysis was validated on a continuous basis within the research team, but also with Royal Society staff.

2.6 Initial findings of this report were shared with invitees to three complementary webinars in late April 2021, comprising in turn a) mathematics academics, and computer science and data science specialists; b) specialist and non-specialist users and policy personnel; c) teachers, educators and education researchers. Each of those split into three focus groups, and the data from the nine focus groups analysed in similar ways, for both validating and complementary views. A formative discussion was then held with the MF Board. This final report is the outcome of that process.

2.7 It inevitably presents a selective and subjective narrative: there is a wide range of valid contributions to add to the debate, but any overarching account of responses needs to make choices about the relative role given to, and the communication of, those views if the report is to best support the refinement of the Mathematical Futures Board's thinking, and the next stages of the project.

2.8 The main focus of this report is on exploring the qualitative views submitted by respondents. It should be noted that the number of people or organisations putting forward a view in an open consultation is not a valid metric for weighing up qualitative evidence. Our analysis takes into account the broad weight of argument (sometimes expressed tangentially, or in other contexts, in answers to other questions) and of the person or body offering any given view: their expertise, membership size, experience, and other factors need to be taken into account. Clearly views of large organisations representing multi-membership need to be given due weight, but this does not diminish the importance of individual responses. These are quoted where they provide a contrasting or illuminating comment. However, we also include in Appendix 2 our supplementary quantitative analysis by number of responses discussing an issue (as opposed to number of mentions), used to validate the balance of qualitative findings presented.

2.9 Further details of the methodology adopted, including the approach to research integrity and trustworthiness, may be found in the accompanying technical note. However, the analysis of any such qualitative data is interpretative in nature. Both the authors are very much 'insiders' to mathematics education in England, with attendant both affordances and constraints. Steps taken to minimise the risks inherent in that are detailed in that technical note.

3. Core questions:

The two core questions of the Mathematical Futures Programme are:

- *What mathematical competences will be needed for society to thrive in the future?*
- *How should education systems develop these mathematical competences?*

In the Call for Views questionnaire, these questions were broken down as

1. *Which mathematical competences are most useful to you (or your employees) and why?*
2. *What are the most useful mathematical competences that citizens need now and why?*
3. *Do you think the nature of mathematics, and the role it plays, have changed over the past twenty years? If so, how?*
4. *Thinking about the needs of citizens, how should mathematics enable the next generation to participate in society?*
5. *What should be the main goals of mathematics education, and why?*
6. *What do you expect to be the challenges facing mathematics education in the next twenty years?*
7. *How could the challenges you have set out in your response to the previous two questions be addressed in practice?*
8. *What else should the programme consider in order to answer the two core questions?*

4. Responses and emerging overarching themes

4.0 Overview of responses

4.0.1 In this chapter we first give an overview of the thrust of responses, their 'shape' and profile, before then considering responses to each question in turn. We make considerable use of quotations, not only to elaborate on reported findings, but to illustrate how responses are being interpreted, the range of sources and the depth of responses. Throughout, there was an underlying challenge with the vocabulary used around computing and digital skills, as well as in relation to 'data science'. 'Digital skills' usually seemed to relate to skills in harnessing digital tools for a variety of purposes, whereas 'computing' appeared usually to encompass computer science (knowledge of systems, coding and computational thinking, communications and hardware), plus knowledge and application of generic and specific pieces of software…. . 'Data literacy' was commonly treated as lying within a broad definition of mathematics and allied to statistical literacy, whereas 'data science' was usually implicitly more interdisciplinary in nature. However, such terminology was often either implicit or not entirely consistent between responses, so there is a degree of uncertainty around the relevant reported responses.

4.0.2 Most respondents focused on recent and emerging trends, with only two or three disruptive 'blue skies' responses, discussed later. Where there were, unusually, responses challenging the prevailing thrust of views, those are also noted. There was little appetite expressed for a fundamental rethink of the nature of mathematics, or of mathematics education, but usually, an expanded and re-focused one, with an enhanced role for creativity and overarching productive dispositions:

Curiosity, adaptability, courage, imagination, judgement and growth…"Out of the box" problems (the Covid-19 pandemic, climate risk, the financial crisis…) are occurring with increasing frequency. "Out of the box" problems require "out of the box" solutions (Education Endowment Foundation).

4.0.3 Fundamental to the range of responses and to answers to several of the questions, was a conviction that as well as core numeracy (and a particular focus on financial literacy), developments external to education mean that we have now moved to a position where an expanded view of mathematics education should incorporate core data and statistical literacies (though the position of a wider 'data science' education was more contested), and that all such education should make significant use of digital, as well as traditional, tools for those mathematical purposes. Such views underpin much of what is discussed in more detail below.

4.0.4 Many respondents did not clearly distinguish between the questions asked, or else found those difficult to answer, so that some responses were repeated across questions. This might be symptomatic of the genuine difficulty of the questions asked, a perceived need to emphasise a particular response, and/or the limited time and energy available to answer the questionnaire. Because of the misleading nature of quantitative reporting of responses, or mentions, in what follows we usually refer to 'many', 'some', 'few', 'handful' to represent decreasing degrees of discussion, though somewhat weighted towards groups or organisational responses. Whereas, as can be seen from Appendix 2, it was not common for *any* area to be mentioned in a majority of responses, there are a number of important areas where a view was promulgated in a wide range of responses, and there was also little counter-narrative. We refer to these as 'views that received wide support', although that might have varied in depth or strength, but below, we also point to any small but perhaps significant competing views. A small number of areas attracted a range of views, and again, we point to that below.

4.0.5 Responses around the mathematics used, or needed, often focused on surface level features, rather than digging more deeply to identify e.g. the algebraic thinking implicit in spreadsheet use, or the proportional reasoning widely used in everyday life: the literature[2] shows many users appear to operate without full awareness of the mathematical skillsets on which they habitually draw.

4.0.6 In general, we were surprised that contributory views were remarkably consistent across different categories of respondent, though often expressed with different profiles. Appendix 2 does not therefore distinguish by respondent group. As might be expected, computer scientists were overall more likely than other groups to make comments about the use of programming or computational or algorithmic thinking for mathematical purposes. Academic mathematicians and computer scientists saw the importance of developing higher level mathematical competences, though they did not expand on that, and making cross curricular links and applications. All respondents reflected something of the mathematical needs of their own work, though no academic mathematician was specific about the elementary mathematics on which their own trajectory depended, so any particular needs of those who will become intensive users of more advance mathematics were not explicitly represented.

[2] E.g. Kent, P. & Noss, R. (1998). The visibility of models: Using technology as a bridge between mathematics and engineering *International Journal of Mathematical Education 31*(1) DOI:10.1080/002073900287390

4.0.7 Those professionally involved in education were more likely to make detailed comments about curriculum, pedagogy and assessment, while those who were using mathematics in their jobs focused on the importance of improving mathematical, financial and statistical literacy and being able to apply what is learned in real contexts. All groups stressed an increasing need to educate for problem-solving, critical thinking, collaboration and reasoning that use mathematics. They also saw the importance of further improving attitudes to, and confidence with, mathematics.

4.0.8 All groups perceived a need for greater (quantity and quality of) investment in mathematics education, particularly in the recruitment and retention of well qualified mathematics teachers. There was a quiet voice across all groups to increase diversity of participation and ensure greater equity in mathematics education, and this theme probably merits further work. Any difference in the themes inherent in comments were largely small, though the profile and balance differed across responses, often naturally reflecting particularly areas of interest or of expertise. The nine webinar focus groups served both to validate the thrust of emergent themes, and to enhance or enrich those, except in a small number of cases where they reiterated or expanded existing disagreements. Such instances, and significant divergences in previous data, are identified explicitly, and discussed, below.

4.0.9 A few particularly reflective responses, including additional contributions, began by addressing core issues they felt were not asked for in the questionnaire: some felt it was important for the MFP to go beyond current or future economic 'need' to consider wider purposes of education beyond 'goals':

> *Ultimately seek to improve the way we live our lives according to the fundamental principles and foundations of society: moral, equity, justice, health, economics (Academic mathematician 6) -*

and we return to such discussion later in chapter 5. Others, particularly those from sponsors of MFP and other employers looking for well-developed mathematics and/or digital competencies, were robust in making their case that 'doing nothing' is not an option: the UK supply of home-educated employees with sufficient mathematics skills is inadequate, is a compelling priority, the need will grow, and dependence on overseas-educated workers is a short-term 'patch' that does nothing for national longer-term security or thriving. Such sentiments went well beyond obviously mathematics- and digital-intense employers:

> *Currently, there are few roles which do not require (mathematical) upskilling. The learning curves required to use large scale data analytics systems are significant for individuals who lack strong mathematical skills. Basic numeracy as well as employability skills such as mathematical reasoning, problem solving and working collaboratively (are needed)… Paucity of mathematical competences is seriously impacting growth and will do so further.. (our) scientists are expected to have data analysis and data visualisation skills. However, it is challenging to find applicants with the required skills set in mathematics…Employees need mathematical skills ranging from basic skills (e.g. data manipulation) to more sophisticated skills (e.g. building machine learning models)…. we find it difficult to recruit staff with the correct competences (e.g. data science) (GSK);*

> *Although it may be possible to recruit into data analysis roles, it is challenging to find staff who are capable of 'lifting the lid' on mathematics. In England, it is very challenging to recruit engineers or scientists with the prerequisite mathematical skills, and this should now be a priority (ARM);*

> *The impact of inadequate numeracy and data skills in the workforce has a big, and growing, impact (BBC).*

And this extends at least to Scotland:

> *There continues to be a skills shortage in basic and complex numerical skills, solving complex problems, basic and advanced ICT skills, and the ability to prioritise and manage own tasks, according to https://www.gov.scot/publications/future-skills-action-plan-scotland-evidence-analysis-annex/pages/7/ (Scotland coordinated response).*

Such evidence underlies the first widely-supported key theme:

Key Theme 1: Individual and societal mathematical needs are already not being met

- There is compelling evidence that limitations in UK mathematical skillsets at all levels are already limiting economic and societal thriving, with activity in related fields often maintained by overseas-educated workers. This situation may not be sustainable and demand is likely to increase.
- Individual and societal thriving increasingly depends on all experiencing, and mastering at an appropriate level, mathematics as a creative, fallible, sometimes-demanding but often empowering route to interacting with the world around us.

4.1 Mathematical competences thought to be most useful to respondents or their employees

4.1.1 As might be expected, this question produced a wide range of responses, that usually focused on the obviously mathematical actions inherent in a role, for example 'using a spreadsheet' but often did not consider the implicit use of mathematics, or the sort of mathematical thinking on which such use builds – for example the proportional reasoning used, or the algebraic thinking or unseen mathematical models harnessed. Kent and Noss (op.cit.) problematise these 'visibility' issues since they lead to a lack of valuing, and critique, of the underlying mathematics.

4.1.2 Responses from *academic mathematicians* were generally at a high level: 'formulating and solving mathematical problems', 'logical thinking', 'mathematical thinking', 'reasoning', 'modelling', 'ability to communicate mathematics', 'abstraction', 'generalisation', 'basic numeracy', 'statistical and data literacy'.

4.1.3 *Computing professionals* quoted 'problem solving using number, algebra, stats, data analysis', 'reasoning about and applying mathematics', use of both discrete and continuous mathematics, software specific skills, logic, abstraction, and in particular knowledge of data and set theory, functions, estimation, algorithms. Some responded more strategically:

> *Algebra, as it's the toolbox to translate real world challenges into a mathematical framework.... Statistics - as it's a way to describe and understand processes in the real world.... Calculus - to describe change at arbitrary scales and to find 'best' solutions in complex, high-dimensional scenarios. Data Science - joining the above ... the way to create actionable insights in a mathematically sound way and go beyond what the human brain can extract due to the brain's time and memory limitations (Computer professional 10).*

4.1.4 *Specialist users of mathematics* often offered quite detailed and specific answers, implicitly or explicitly raising some clear issues for mathematics education. They talked about the centrality of 'solving real life problems mathematically, critical analyses, mathematical modelling, approximate methods, apply statistical tools', reasoning, data analysis, 'mathematical mindsets'. They pointed to branches of mathematics theyused heavily – variously algebra, differential equations, statistics, big data, probability – but also the mathematical mindsets and collaboration that needed to be drawn on:

> *Formulating problems mathematically: it is essential to be able to solve difficult engineering problems by analysing them and breaking them down into individual parts, which enables the application of standard mathematical tools and techniques to solve the problem (Specialist user 7);*

> *Thinking mathematically; being confident at analysing numerical data; the ability to learn new mathematical techniques, and to search the literature for new ones that might be useful; problem solving; experimental design; algorithm design; continuing to think logically whilst those around are panicking or engaged in groupthink (Specialist user 4).*

4.1.5 *Users of mathematics* generally pointed to deep and flexible understanding of number, including proportional reasoning, measure, data handling and statistics, and some had considerable need for 2-D and 3-D geometry. They pointed to a need for mathematical reasoning, and to a widespread use in their fields of digital literacy and of a range of basic IT tools – 'confidence with mathematical concepts, data analysis, digital tools such as Excel; communication' – but did not, for example, identify the algebraic thinking underlying much spreadsheet use, and most did not talk about the mathematical modelling implicit in their responses. They identified the value of embedding mathematical knowledge and thinking in new contexts across their fields of work.

4.1.6 *Computer science educators* pointed, unsurprisingly, to computational and programming skills, within a problem solving and modelling digital environment. Many *mathematics educators* cited a need for a sustained and effective use of problem solving and reasoning, entailing flexible thinking and positive mathematical dispositions, and primary educators often focused on core conceptual grasp of elementary mathematics.

4.1.7 *Educators* said teachers need to: understand the nature of mathematics; have knowledge of mathematics curriculum; of misconceptions, of relationships between concepts, language, mathematical models and visualisation – in other words, well beyond familiarity with curriculum content; also 'skills to understand the world' - data, mathematical modelling, statistics, communication skills, confidence. For science, teachers and students need numerical computation and sense-making, including of large/small numbers, understanding of probability, statistics and risk, simulation, algebra and graphs:

> *I regard quantitative skills and statistical/data understanding as being central to young people's development and preparedness for adult life. The ability to gain competence with early algebraic concepts (is) frequently a good indicator of mathematical success later on (Teacher 9).*

In summary,

Key Theme 2a: Mathematical competences needed are changing

Respondents reported a range of mathematical competences important to them in their work. They commonly identified deep and flexible numeracy, statistical understanding, problem solving and reasoning, modelling and increasingly, use of digital and data-related skills, together with some role-specific competences.

4.2 Mathematical competences currently needed by citizens

4.2.1 Across respondent groups, answers to this question largely focused on universal needs - basic numeracy and financial literacy, inclusive of measurement, 2-D and 3-D spatial, and graphical information, with supplementary skills for those who want or need to go further. Many respondents explicitly included the sense of magnitude of number, estimation, prediction and sense-making. Understanding of statistics, analysis and interpretation of data, and elementary probability so as to understand risk and bias, were also thought to be universal needs:

If the public could have a better understanding of where quantities (e.g. £5M per annum) need a context (e.g. from a £50Bn budget) and/or where proportions can be misleading (e.g. less than 1% of £50Bn is still several hundred million) (Mathematics user 4);

Recognising that an assessment of a hundred-year risk of flooding doesn't mean you've got 99 years before you need to worry (European Bioinformatics Institute);

There are many benefits to building quantitative skills in the UK population, including helping citizens to participate more fully in the democratic process; enhancing research in universities and in the workplace; and supporting the economy, taking advantage in particular of the advent of "big data" (The British Academy);

Citizens need to be able to critically reflect on the validity and reliability of data they are presented with, in order to make informed decisions (Scotland coordinated response).

4.2.2 Mathematical processes and positive dispositions were also widely valued: 'how to think', reason, solve novel problems, flexibility, creativity, confidence, communication and collaboration with basic mathematics, and being able to relate that to the world around them, being a 'user of maths'. There was some appetite for all having a grounding in computational thinking, and harnessing digital tools and very basic programming for mathematical use. The British Computing Society argued an increasing both *breadth* and *depth* of need:

We believe that the near-universal application of computing has significantly raised the bar on the level of understanding needed.

4.2.3 Since the list of 'essentials' produced was typically quite long, it's highly likely that many responses represent respondents' priorities, or those aspects they consider new or in danger of being missed, rather than comprising an exhaustive list of their preferences. For example, not all responses talked about a need for robust basic numeracy, even though their later responses assumed that.

4.2.4 Some employers were very specific: for example, some in the pharmaceutical industry quoted a wider set of mathematics competencies, including linear algebra, differential equations, algorithms and computational mathematics.

4.2.5 One free-form response identified a depth of not always obvious mathematical knowledge needed for developments in computing:

Evidence suggests that careers in AI benefit from a more advanced mathematical knowledge such as predicate calculus and graph theory; and that knowledge of calculus supports system engineers…..Graph theory knowledge is needed for network routing; data encryption and data compressions both involve mathematical functions and transformations. Graph theory also links *with software engineering. A good understanding of sets is helpful for relational database tables (England government department).*

4.2.6 Mathematics educators often 'unpacked' the supporting characteristics and knowledge needed for such competences, noting that mathematical reasoning and problem solving requires 'resilience and perseverance', mathematical models need setting up and their limitations appreciated; that learners need to make connections within and beyond mathematics, with the curriculum more embedded in meaningful applications, including global issues:

Advances in digital technology shift the balance between skills and knowledge to conceptual and technical grasp of mathematical ideas (MEI);

Collective knowledge and critical understanding (are) needed to promote human rights, cultural diversity, social justice and equality. Mathematics should play a central role in a school curriculum designed to address the economic, social and environmental challenges faced by the global society (Mathematics educator 9);

Understanding how data are used,… including: in algorithmic decision making…; in measuring things such as health, inequality, deprivation, environmental impact; in national finances (deficit, debt, inflation) (Mathematics educator 22).

4.2.7 Teachers were particularly aware of the need to inculcate confidence and meaning-making in the use of mathematics, and teacher managers were typically especially focused on the need to deal with 'real world' applications, and to critique sources and meaning of data in confident and informed ways:

Classification, comparison, and questioning the meaning of source data, how it has been derived or acquired, and alternative interpretations.,… Questioning received wisdom, asking about the underpinning principles, developing one's own mental model of how different processes play out and communicating about them with others (Teacher manager 3).

4.2.8 It is important to note that such needs appear to spread across much of the UK, with Scotland also experiencing a skills shortage in basic and complex numerical skills, solving complex problems, basic and advanced ICT skills for such purposes, and the ability to prioritise and manage own tasks, according to https://www.gov.scot/publications/future-skills-action-plan-scotland-evidence-analysis-annex/pages/7/ :

With the abundance of information available to us, being able to process, analyse and evaluate this in order to solve problems will be even more important. Weighing up conflicting arguments using logical thinking tools and being able to make use of these tools in a variety of contexts will enable us to do this in complex, ever-changing environments (Scotland coordinated response);

4.2.9 Respondents described a need for researchers who can deploy and integrate both quantitative and qualitative research methods, as well as individuals sufficiently confident with the technical aspects of developments such as artificial intelligence, to apply their social and human expertise in shaping the legal, moral and ethical frameworks required to underpin the 'new digital age'.

This section is summarised together with the next, on page 16.

4.3 Has the nature of mathematics, and the role it plays, changed over the past twenty years?

4.3.1 There was negligible response that the fundamental 'nature of mathematics' has changed – 'Prove a result today and it will still be true in 1000 years' - but rather, that the ways in which it is applied, the tools it employs, what citizens need to know, and the role of mathematics in employment and everyday life, including its public exposure, have changed. There is consequently a change in how mathematics is perceived, applied and its growing importance to work, life and policy decisions (though three responses said that the increased role of mathematics in public life has made some of the pubic continue to feel that 'maths is too difficult').

4.3.2 Respondents identified that mathematics and mathematical modelling are now required in all study of biology, economics, sociology, the humanities, health ('we can model real world situations, ask real questions'), and computing technology has opened a wider range of mathematics to be used and understood by the general public: 'daily data updates around the pandemic and discussion of risk' 'data and use of algorithms for decision making with bias'….

4.3.3 A few respondents felt that there have been changes in the nature of mathematics, though others conceptualised the same changes as those of tools for mathematics, or the continuing expansion of a 'live' discipline. They identified, variously, the role of machine learning and so artificial intelligence, the use of algorithms in decision making, use of technology for mathematical exploration, proof theory and category theory, the use of topology for data analysis….

> *Technology is often an enabler to mathematics, allowing it to have more of an impact, and there have been a lot of recent technological developments around 4/5G, smartphones, edge computing, machine learning optimised graphics cards, the internet etc (Computing professional 6).*

4.3.4 Some argued that the nature of proof itself has changed, while others pointed instead to new technologies supporting the visibility of routes to proof, for example (as in e.g. Christou et al., 2004).

4.3.5 The catalyst behind such changes is the growing availability and use of computers, which have made large scale data available for 'rigorous' analysis and should 'make us recognise how algorithms dominate decisions in our life'. Data science has acquired great status, and the mathematician's role is then about judging what to ask the computer to do, rather than doing it; the implications are profound:

> *Maths is increasingly integral to lots of other scientific fields. This, along with things like the rapid growth in AI, means that (applied) maths and operational research are having an increasingly direct (and sometimes quick) impact on society. (The Operational Research Society);*

> *Data has become 'the new oil' - a valuable commodity. Most obviously for consumer targeting with advertisement etc, but increasingly in other domains too. This has left no option but for businesses world-wide to adopt a data-driven approach and put data at the heart of their operations (Specialist user 3);*

4.3.6 Mathematics educators argued the pandemic has shown the need for competence in interpreting statistics and graphical representations, understanding and calculating risks – and so, importantly, in determining responsible collective and individual responses including their ethical and moral dimensions.

Equally, in parallel with 'exponential increase in the availability and use of data', a significant number claimed that while pure mathematics has not fundamentally changed, the applications of mathematics have become much more widespread and data-intense.

4.3.7 Unsurprisingly, teachers often pointed to the consequences for mathematics education, with a much reduced role for learning formulae or performing calculations - 'you don't need to be able to calculate a T-Square or a correlation coefficient' - but citizens do need to have the underpinning conceptual understanding so they can choose and interpret appropriate use.

In summary:

Key Theme 2b: Mathematics, and mathematical needs, are evolving. Changing roles for mathematics and the tools available have transformed its potential impact and the scale of need.

- There was wide agreement that mathematics has not fundamentally changed its nature in recent years – but that the roles it plays, and the tools available for mathematical activity, have expanded considerably with the expansion of technological capacity and access, and explosion of easily-availability data.
- As a consequence, mathematical aspects of work can have more and wider impact, and their roles are more often in the public domain. Examples given were the developing use of algorithms for decision-making, role of machine learning, and expansion of applications well beyond the sciences into social sciences, arts and humanities, including sometimes questionable use in the media.
- Mathematics has, however, continued to evolve, e.g. in category theory, and applications will continue to change as AI and technology expand further. As a consequence, mathematical work can have more and wider impact, and its roles are more often in the public domain.
- There is therefore a substantially enhanced need for mathematical equipping at all levels.

4.4 Enabling the next generation to participate in society

4.4.1 In answer to this question, many respondents focussed on how mathematics should be taught and learned to enable 'the next generation to participate in society', interpreted as needs beyond the workplace (there has for example been some USA work on developing mathematical needs focused on 'citizen needs' vs. 'worker needs'), and some referred back to wider current needs identified in earlier responses. Overall, responses suggested it was anticipated that present trends in needs would continue, including widespread need to 'Interpret and critique data, and especially, appreciate the difference between correlation and causation'; that the impact of digital tools, increasingly available data and mathematical models on our lives would continue to increase, and so there was a wide need for a deeper, more confident mathematical equipping. There were also wider claims for mathematical competences:

> *Mathematics makes or helps to make people literate, competent, independent thinkers and problem solvers, more critical, more analytical in their approach of situations, more robust against mass phobias and less likely to accept misconceptions and trust false prophecies. It helps people think more clearly in areas outside mathematics as such (Academic mathematician 13);*

4.4.2 Although basic mathematics literacy (defined broadly) was thought to be important, a pervasive thread was the need to develop mathematical *processes and productive dispositions*: deductive and inductive reasoning rather than memorising, critical thinking rather than passive receipt of facts and procedures, deriving rather than calculating, clear and rigorous mathematical communication; developing positive attitudes to mathematics learning – persistence, a 'can-do' attitude, and a sensitisation to, and wisdom with, mathematics around us:

> *People's understanding, at least in principle, of how mathematics through computing shapes the modern world…. (enables citizens to) understand at some level the importance of using evidence to support decisions and how that evidence should and should not be used (British Computing Society);*
>
> *Mathematics should enable future citizens to become part of an informed debate, accurately interpret datasets and to not be manipulated by the misuse of statistics (BBC).*

4.4.3 Allied to such outcomes was a wide espousal of a conceptually-focused mathematical and financial literacy, including data, statistics and probability literacies: citizens will need a sense of size of number and of risk. Although there was an uncontested appetite for 'data literacy' to be considered an integral part of mathematics, some responses referred to an ill-defined 'data science' with a less clear status.

4.4.4 Some in webinars went further, talking about 'a broad mental maths' for everyday personal and employment functioning, that goes well beyond getting an answer: the Northern Ireland curriculum focus on such competences was cited. For more complicated tasks, future generations may not need to carry out repetitive complex calculations, but they will need to be able to choose appropriate processes that may be carried out by machine, and critique and interpret both input and outcome. They will grasp how mathematics underpins many areas of life and will have confidence to apply mathematical competences.

4.4.5 It was widely argued that students need substantial engagement with authentic data through much of their school lives, using appropriate software, if they are to become confident in the interrogation, critique, representation, analysis and interpretation of data. Overall, it was widely thought that the use of digital tools for mathematical purposes should be a central part of students' mathematical experiences though, with one exception, respondents thought that core computer science and IT competences need to be kept separate from mathematics as curriculum strands. Although there was some small-scale mention of the desirability of developing 'computational thinking' it was not usually clear what was meant by the term and we suggest more work could usefully be done to explore this area.

4.4.6 In one webinar focus group there was a lively debate around the relationships between mathematics and computing education and in others there was appetite for cross-curricular and project work, including in primary schools, that developed a range of mathematical and digital competences in meaningful contexts. For example, one contributor talked about the EU funded *Make it Open*[3] which is an international collaboration based on the recognition that:

> *Policy makers and practitioners need to find new mechanisms, processes and tools that will help schools rethink learning boundaries and equip future citizens with the right skills to critically evaluate sources of scientific news, make fact-based decisions and informed choices.*

4.4.7 It was commonly argued, therefore, that in a digitally-pervasive world, citizens should be equipped to understand the potential, and limitations, of digital tools for mathematical purposes. In so doing, their digital literacies would be strengthened. Consequently, all citizens would need basic mathematical, data and

[3] https://makeitopen.eu/about/

financial literacies, rooted in confident conceptual grasp and robust processes that include appreciation and experience of how technology can support mathematical processes.

4.4.8 Respondents also widely thought that such equipping could only be acquired widely via experience of multiple real examples: 'probability through weather; exponential growth through social media going viral; experiencing geometry through collaboration between art and design - maths as creative and social enterprise'. Two aspects of this were valued: exposure to important applications of mathematics carried out in other disciplines, and coming to engage themselves with authentic opportunities to develop as *users* of mathematics. It was thought such approaches would create confident and flexible citizens who can solve problems, including collaboratively, moving on from 'maths is too hard for me'. (It was also argued that as such applications expand, different mathematical skills might be needed.)

> *I think there are two separate things most people need to get from mathematics. The first is numeracy. … the standards here are already pretty good. The second is the mathematical approach to problem solving, which I don't think we teach very well (Computer professional 17);*
>
> *Active and scientifically literate citizens use mathematics to make informed decisions on their individual actions and behaviours, understanding the consequences of these decisions and contributing positively to community and wider society activities in addressing the challenges of our generation (The Association for Science Education).*

4.4.9 However, in questionnaire responses there was a small but significant voice, echoed in webinars, cautioning that focusing instead on making sense of basic mathematical knowledge, concepts, interrelatedness and skills would enable coherent mathematical conceptual frameworks, whereas an early applications-based approach would jeopardise secure mathematical foundations.

4.4.10 Implicit in many responses is the need for many young people to learn mathematical knowledge and skills that go beyond the basic equipping needed by all. Some webinar respondents moved on from this, arguing that, paralleling literacy education, it is important to be able to 'read' and critique mathematics beyond what one can do for oneself, and to consume arguments and ideas that are not (at least yet) fully understood.

We summarise this section, together with section 4.5, on page 22.

4.5 Main goals of mathematics education

4.5.1 Many answers to this question repeated what had been stated in questions 2 and 4:

> *Mathematics for the informed consumer, citizen and employee (BBC);*
> *To encourage and train our students to be curious, adaptable, courageous and imaginative, giving them a growth mindset education….Equip learners with skills, engagement, motivation, and confidence to study subjects requiring maths skills and enter careers necessary for our society and economy as a whole to thrive (Institute and Faculty of Actuaries).*

4.5.2 There was a high degree of consistency in the answers across groups of respondents: all citizens need basic mathematical, data and financial literacy, rooted in confident conceptual grasp and robust processes that include appreciation and experience of how digital tools can support mathematical processes. A small minority suggested that coding/programming should be an integral part of mathematics education so that skills and techniques from computer science support and enrich use of mathematics, but this was not widely seen as a priority in either survey or webinar responses (and discussion of how mathematics education should contribute to computing competences was negligible).

4.5.3 All students should have the opportunity to study mathematics at least up to the level they will require for future study and work. Webinar participants suggested that there was also a place for exposure to horizon-extending and enriching mathematics that would deepen students' understanding of mathematics as a powerful tool for engagement with the world around us. For example, thinking about infinities of different sorts of numbers does not need a rigorous exposure to number theory; the idea that imaginary numbers underpin a theoretical grasp of electronics, and so digital technologies, is mind-blowing to many teenagers, but does not need to entail a deep understanding of complex numbers.

4.5.4 Expansion on these views often addressed needs at three different levels – though later comments often implied there was in fact a continuum, and some branching, of mathematical need. Several academic mathematicians, for example, suggested:

- Provide sufficient mathematical skills for the non-specialist, so that they have both an appreciation of the power and importance of mathematics and also the ability and inclination to use mathematics to enhance their own lives. The related competences were widely thought to include a core and confident numeracy (including spatial and algebraic literacies), financial, data and statistical literacies - including exploration and support for those via a variety of digital tools.
- For many scientists, technologists, economists, decision analysts, etc., provide a wider solid training in mathematics that goes beyond just useful methods, but also provides an idea of the importance of mathematics as a key part of human culture.
- For mathematical scientists, more extensive mathematics education should cover 'pure', 'applied' and 'applicable' mathematics. For this group, abstraction is particularly important, as is learning to engage with complex, unstructured problems, including collaboratively and with digital tools.

Such a range was supported across groups, though framed in different ways.

4.5.5 Providing for this range of needs was thought to require a variety of flexible mathematical pathways, some of which might, particularly later in secondary education, focus on particular aspects of mathematical functioning – say data literacy, or computational mathematics, or a combination of pure, applied and applicable mathematics. A key issue here, then, is who makes those choices of pathways, on what basis, and at what stage.

4.5.6 Many academics and others thought education should use computer software and, sometimes, programming to enhance understanding in mathematics, and help students appreciate rapidly developing technological potentials. Four individual respondents thought mathematics teaching and learning should include the use of, say, Python or R. Several responses however pointed to the necessity of ensuring a focus on essential mathematical skills, knowledge and processes is not bypassed in the drive to harness digital tools:

Introduce basic Python (using the Raspberry Pi) to all school children…Give teachers one day per week to learn Python programming as part of CPD…Ensure that the basics of mathematics are not replaced with technology (Computing professional 16).

In-depth coding was not commonly embraced by other groups, beyond its direct application to school mathematics, although there was a moderate appetite for elementary programming using Scratch or Logo for geometrical purposes within primary and/or secondary mathematics, and it was thought such use would also support the development of computational, including algorithmic, thinking.

4.5.7 However, as one mathematics user argued, getting the *right mix of mathematics and computer science* will be tricky and crucial. For example, what is the overlap and how should that be experienced? How can we encourage students to take both subjects as a pair and make the experienced sum of the two subjects greater than the sum of the parts? - and so on. Some such comments seemed to stem from multiple changes to the approach to computing education in England in recent years.

4.5.8 Responses here also often reiterated a need in mathematics education to focus on the *mathematical purpose* behind the use of digital tools, and to continue to complement that with core mathematical work that includes 'by hand' operations and experiences that support conceptual growth:

It's difficult to teach people how to use a map and compass when they have GPS but it still gives a deeper understanding. Don't let computers and what they can do mask the need for understanding of core mathematical skills. But they can bring a lot to enhance and make it interesting and relevant (Wycombe Abbey)

There was (very) occasional disagreement: *'computational complexity could be taught in mathematics, for example' (Webinar).*

4.5.9 Respondents from a range of groups thought it was important to proactively address broad mathematical processes and positive dispositions to learning mathematics. Mathematics educators analysed this as

This means we develop mathematical thinkers who make, explain, justify and compare decisions, take a flexible approach using what they know to problem solve and derive solutions, act creatively looking for connections and building logical pathways. Although we want learners to have the skills for life and employment…they need analytical skills to navigate the complex world. But this involves strengthening a positive attitude and increasing motivation for all students to want to develop their mathematical thinking, ….and reduce their anxiety. We need to end the acceptance that it is OK to say 'I am no good at maths!' (Mathematics educator 1)

4.5.10 Mathematics users and others emphasised that confidence and mathematics self-efficacy must be developed and nurtured from the early years, and such messages were reiterated in webinars: 'Why is it still acceptable to label children as mathematical failures from the age of five?'. Teacher managers often gave a non-specialist but educationally-informed response, including at least five of nine such respondents emphasising thinking and reasoning logically, developing confidence and fluency, creativity in problem solving. They, and others, pointed in low-key ways to a perceived need to enhance diversity, equity and inclusion in mathematical attainment, participation and work, btu such arguments were not explored in depth: we suggest there is a need for further work in this area.

4.5.11 There were pleas, especially from those in education, for strongly-communicated messages of 'make maths enjoyable; its relevance for practical living; everyone can do mathematics', 'love mathematics', 'appreciate the beauty and logic of mathematics', 'increased confidence in mathematics', 'make maths cool', 'The goals are to inspire the next generation' - but also that mathematics is a 'way of thinking' and not just a useful tool'. Respondents felt such beliefs need to be nurtured from early years, opening up the learner to the 'magic' and 'rich world of mathematics'. This will require, inter alia, the provision of resources that help teachers teach mathematics as creative:

> *To help students see the beauty and creative potential of maths (Mathematics educator 19);*
>
> *Abstraction is a key skill which involves reasoning and helps children make links across concepts. But it stems from engagement with the tangible ….or interacting with computers (GSK);*
>
> *The most important thing is to cultivate mathematical ways of thinking. This comes from an interest and engagement in a problem you're interested in solving, so that students go to the maths classroom with a curiosity and a need to understand. In that way they acquire …fascination and delight in the subject (Computer science educator 1).*

4.5.12 Across responding groups, there was some mention of a need to embed mathematical activity across any curriculum: 'there's a need for 'a 'humanities' angle in maths, including post-16' 'Mathematics should be part of every educational curriculum at school, college and university'; 'embedding quantitative skills across the whole curriculum, rather than seeing mathematical competences as standalone skills'. One webinar focus group noted that links across the curriculum, and to the outside world, should be made overt, and tangible – 'why might it be important to learn this?' Soberingly, the Geographical Association work on climate change perceptions was reported to show teenagers don't think mathematics has relevance to tackling such issues, and especially for girls, the mathematics conversation needs to be about more than just about STEM.

> *A main goal for school mathematics education is to open children's eyes to maths through the processes of "mathematising" - that is, to pose and solve problems that use "real", "pseudo-real" and pure mathematical contexts (Mathematics educator 2);*
>
> *We should be developing mathematical thinkers who make, explain, justify and compare decisions, take a flexible approach using what they know to problem solve and derive solutions, and act creatively, looking for connections and building logical pathways (Mathematics educator 12).*

4.5.13 Further, a few respondents articulated an argument that there is a lifelong need to have access to mathematics education, including beyond a person's early goals, so as to achieve aspirations, enrichment and empowerment in personal lives, in education and employment and in wider society – and because technology and uses of mathematics will continue to evolve. One funder questioned the balance between school education and education for role-specific competences within the workplace, and argued both are needed.

> *Citizens need lifelong access to high quality engaging online learning so they can gain maths competences at their own rate …If we focus on enabling learning, we enable citizens (Computer science educator 5);*

4.5.14 Although beyond scope, one education policy organisation argued for 'a re-focus in universities to provide a more effective education for people who will go on to work in the field rather than at the frontiers of the discipline'. The British Computer Society, and a small number of computing professionals, also argued that a central focus of the project should be a meaningful debate about what an outstanding education in each of mathematics and computing looks like, and how each informs and enriches the other.

> *I think that every citizen needs an elementary grounding in computing (more specifically computer science), in the same way and for the same reasons as they need a grounding in maths and natural science. It gives them control, agency, and informed judgement (Computing professional 23).*

Summarising these two sections together then:

Key Theme 3: Mathematics education should therefore be expansive, but probably separate from computing education

- Mathematics education should equip all with basic mathematical, data (including statistical) and financial literacies, rooted in confident conceptual grasp and robust processes that include appreciation and experience of how technology can support those. Wider data science education might take place across the curriculum.
- Problem-solving, creative and critical thinking, reasoning and abstraction, underpinned by core knowledge, are central to mathematics. Young people need an appreciation of the power and importance of mathematics as integral to human culture and also the ability and inclination to use mathematics to empower their own lives. They should experience both satisfaction and enjoyment in their use of mathematics, often working collaboratively and cross-discipline, and develop productive dispositions towards its use, so that all come to identify as successful users of mathematics at an appropriate level.
- Digital skills, techniques and resources support and enrich use of mathematics: for example, confident use of appropriate software supports students in mathematical understanding of authentic data, and of risk. However, almost all thought computing per se and mathematics should be kept as separate but persistently complementary curriculum strands, especially as students mature. Computational and algorithmic thinking and AI, beyond immediate mathematical purposes, had a low profile in responses. Clarifying the language used around computing would support informed debate in related areas.

Key Theme 4: A variety of mathematical pathways would be needed to achieve this

- Young people's engagement with some form of mathematics education (defined broadly) until age 18, should be via flexible pathways and mathematical focus that together provide for the range of mathematical education needs identified.
- Such pathways should educate for at least 1. Mathematically functional citizens; 2. occupational users of mathematics; 3. specialist users of mathematics, including academic and professional mathematicians. Many need solid mathematical competences that go beyond basic mathematical, financial and data literacies, and some also need more advanced 'pure', 'applied' and 'applicable' mathematics.
- All should have opportunity to study to the level they will require for the future, in data-rich and digitally-enhanced ways, and to acquire further mathematics learning as individual and societal needs develop: some such additional learning could also be employment-based.
- There would be a challenge in devising structures that support timely, equitable and successful choices of mathematical pathways.

4.6 Emerging challenges for mathematics education in the next twenty years

4.6.1 Call for Views respondents and webinar participants were outspoken that any such expansive vision requires coherent parallel development of curricula, pedagogies and assessment, together with sustained and substantial teacher re-equipping, and public (and media, and political) valuing of such education. There is considerable and international evidence[4] that if any one of these 'legs' is not coherent with the whole, then aspirations will not be fulfilled: *The strap line should be 'Coherent curricula, do less, do better' (Webinar).* However, effective ways of achieving that while also allowing for flexible pathways and a balanced whole curriculum, are an issue:

> *Education structures, including timetables, currently reinforce early decisions and priorities, including 'what sort of person' you are: we need creativity and boundary-crossers for modern problems, and a willingness to work outside comfort zones: the exciting things happen in 'gaps' (Webinar).*

4.6.2 One of the biggest challenges, widely identified across groups, is the shortage of well-equipped teachers of mathematics, at all levels from early years onwards (with knowledge, skills and affect that includes a deep interest in, and understanding of, both mathematics and learners). Depths of engagement with the issue varied: some argued that if more well qualified (and better paid) teachers are recruited then this will help raise standards, engage more pupils in learning mathematics, improve diversity, and improve attitudes to mathematics. Mathematics users and others talked about a need to find ways to overcome teacher shortage, low pay, low respect, as well as 'providing teachers with professional development for mathematics for the 21st Century':

> *High quality and subject rich teacher professional development will lead to learners who want to develop and go on and study mathematics, and will change negative attitudes (Teacher manager 4);*
> *Specific focus should be placed on the Initial Teacher Education and probationary year phase (Scottish combined response);*
>
> *The importance of paying attention to teachers, not just students, may be the #1 lesson of the history of maths education from the 1960s-2000s (Mathematics historian, webinar).*

4.6.3 Mathematics educators argued that an ambitious and empowering mathematics education needs: teachers mathematically confident and themselves empowered at the required level; a commitment to high quality up-to-date and career-long teacher professional development to sustain a well-qualified teaching force; retention of teachers; make teaching attractive to mathematics graduates. Further, the changing and ever more complex world requires 'availability of appropriate technology and its use, in a way that enhances learning, by well-trained teachers who have time to become familiar with the technology and its uses in the real world, as well as for curriculum purposes'. Some, but particularly those more working closely to education, further unpacked what 'well-equipped' should mean, and we report on that in 4.7.

[4] E.g. Schmidt, W.H. and Prawat, R.S. (2006). Curriculum coherence and national control of education: issue or non-issue? *Journal of Curriculum Studies* 38(6), 641-658

4.6.4 However, responses identified also an important challenge in the confident equipping of non-mathematics specialists to teach mathematical aspects of work across the curriculum, from early years onwards. Some in webinars thought cross-curricular work was comparatively easy to achieve in early years and primary settings, where teachers largely work across the curriculum - the challenge is teachers learning to expose and build on the mathematics inherent in such work.

4.6.5 Specialist users pointed out not only a shortage of mathematics teachers for current classrooms but mathematics teachers who can teach a new statistics- and data-rich curriculum and teachers who have experience and knowledge of the application of mathematics in other subjects: 'we have mathematics teachers who have never worked in the field of software and big data' and we need 'mathematicians who appreciate literature, arts, human nature'. An expansive vision was thought to require a passionate, well qualified teaching body who 'have time and energy to develop their own mathematical thinking'.

4.6.6 Webinar participants also identified the marginalisation of mathematical elements of existing upper secondary qualifications, despite recent aspirations to strengthen those, with teachers and students often lacking confidence to engage. Looking forward, they thought the consequences would increase over time.

> *Challenge: to bring computational and mathematical thinking into other subjects, in ways that make students better geographers, historians, biologists. [NB: not to make them better mathematicians or computer scientists] (Computing professional 23);*
>
> *Students should be enabled to experiment, create and draw synergies across different subjects so that they can see how mathematics can be applied in different contexts (GSK);*

4.6.7 A few respondents thought that greater exposure of the roles of mathematics in society was resulting in more positive learners, whereas others identified a persistent mathematics anxiety, concerning since confidence and competence to engage effectively were becoming more central to personal and occupational thriving. Mathematics in everyday life appears to be more complex as the world becomes more digital, with issues around driverless cars, debt, population growth, side effects from vaccination… Those who can't engage critically and with confidence were thought likely to become progressively marginalised over time.

> *'Wider attitudes and values, particularly of family, and later of other role models, are very influential on young people. It is therefore important to continue to develop mathematically-positive public and media messages that communicate that the use and harnessing of mathematical thinking is for everyone. The report Making Maths Count , published in September 2016, sets out three priority themes for maths education in Scotland:*
> *• Transforming public attitudes to maths.*
> *• Improving confidence and fluency in maths for children, young people, parents and all those who deliver maths education to raise attainment and achievement across learning.*
> *• Promoting the value of maths as an essential skill for every career' (Scottish combined response).*

4.6.8 One response placed societal attitudes at the heart of challenges:

> *The primary challenge here is overwhelming - i.e. the evolution of society away from instilling fundamental value in a STEM education. The most rewarded individuals (financially and in terms of appreciation) are those most capable of generating wealth (sales, marketing, business development*

and/or celebrity). We need to re-inject the importance of scientific and technological advancement for society, and recognise the importance of those who support this…Further, adult education, practical/vocational development and research excellence are three different capabilities with three different drivers. We need to enable establishments to focus on whichever of these areas they have most strengths in, without judging one as 'better' than the other. We also need much more flexibility for students to mix and switch their course development across these areas (Mathematics user 4).

4.6.9 There were, additionally, reservations about central commitment to, or capacity for, more effective mathematics education, at least in England:

The programme cannot be divorced from our capacity to reform and change the way a subject is taught which principally comes down to teaching and school infrastructure: there are challenges with the English education system which may be risk-averse, with limited capacity for experimentation and innovation (GSK);

'… the government does not want a mathematically literate society'; 'Issues are currently (party-) politicised so that statistics are used to mislead'….

Such comments foreground a need to craft a wide consensus for moving forward.

4.6.10 Another challenge is ensuring that a stable curriculum provides everyone with the opportunity to function well in this complex world (Two teachers suggested that constant political intervention in curriculum, pedagogy and assessment has a destabilising and sometimes poorly-informed impact on teaching and learning). A curriculum fit for the 21st century would need time to bed in, and for teachers to learn how to make it work for their learners.

4.6.11 A number of questionnaire and webinar respondents professionally close to education pointed to overly 'high-stakes' assessments, especially if restricted to short timed written papers, actually undermining good quality learning as they squeeze out less-easily assessed aspects of curriculum. They pointed, more widely, to the necessity of curriculum, pedagogy and assessment, being well-aligned:

Assessment is critical and a key driver in what is taught (and how). Ways of assessing need considerable review. The setting of questions needs to allow for a variety of approaches and the communication of methods including both diagrams and words. Regulatory frameworks have to support assessment of those aspects of mathematics we value (Teacher 9).

4.6.12 A related issue around high-stakes assessment is that while they have resulted in fresh aspirations, and opportunities, for some, other learners, from as early as age 4, are labelled 'failures'. Although the issue did not have a high profile in survey responses, a number of webinar respondents believed this is highly damaging at both an individual and a societal level, reinforcing beliefs that mathematical activity is only for some, and that it is quite normal not to 'succeed' in mathematics *'Maths is spoken about as though it is a genetic gift, possessed by a rare few and inaccessible to the general public' (Home nation government department).*

4.6.13 There is a challenge also in structuring opportunities for lifelong access to the continuing education needed to support informed participation in a rapidly-changing world:

How can we bring the adult population up to the level of basic mathematical competence and confidence needed in modern life? (National Numeracy);
We must have opportunities for continuing adult learning in priority fields, in a gender-inclusive way (BBC).

4.6.14 At least four respondents pointed to the considerable challenges associated with finding effective ways to harness the potential of technology to support mathematics learning, whether that is school-led or independent: 'How can we engender confidence and proficiency using blended learning? How can teachers teach remotely with innovation?' 'Technology will have more to contribute – but it's not clear exactly what that offers or what related challenges will be'. We return to this issue in discussion.

4.6.15 Finally, responses to this question included a near-outlier:

I believe that the main challenge for mathematics education is to reach the non-achieving tail, and that the solution lies in making better use of ordinary language and its built-in mathematics (Linguistics user of mathematics).

Emerging messages then, are that

Key Theme 5: Our mathematics education workforce is significantly under-equipped, and there are other challenges

- A central challenge of such an expansive vision for mathematics education would be the recruitment, retention and continuing education of sufficient well qualified and prepared teachers of mathematics, as well as the mathematical equipping of teachers of other subjects. (Compared with many developing and developed countries, our aspirations for mathematics teachers, particularly in England, are very modest).
- There is a need to further develop public perceptions so that mathematical competences are understood to be as achievable, and as central to individual and societal thriving, as literacy is, and digital technologies a natural tool in their use; such competences should be valued and rewarded.
- Other challenges include establishing a cross-party political will to prioritise, develop and sustain the changes needed, in ways deeply informed by collaboration across subject-specific and broader education system experts - and then maintaining an inclusive and stable curriculum that is supported by coherent pedagogies and assessments.

4.7 Suggestions to overcome these challenges

In this section, we address in turn the solutions which were offered to address: the scale of the problem and poor attitudes, curriculum, pedagogy, assessment, and teacher equipping (skills, knowledge and affect).

Collaboration, investment and addressing attitudes

4.7.1 There was widespread agreement that meeting such challenges would require unprecedented commitment, use of evidence, and investment, although teacher recruitment, retention and pay were only occasionally addressed. Users of mathematics proposed learning from mathematics education systems in countries with high achievement. They suggested such systems commonly inculcate basic skills with deep understanding, and high expectations of learners from well-motivated teachers.

4.7.2 Responses commonly argued that to achieve such visions UK governments need to take mathematics more seriously – perhaps appointing a 'minister for maths', increasing spending significantly, enhancing its public profile through a maths laureate, and free online 'quality resources' for all learners.

4.7.3 Respondents frequently showed low key but pervasive frustration at the limited impact of a number of well-evidenced reports and initiatives designed to improve the quality of teaching already available. They often felt the key actions recommended were still appropriate:

> *At primary, despite the recommendations of Cockcroft (1982) and Williams (2008), we still have a teaching workforce who are under skilled and lacking in confidence in teaching elementary mathematics… Initiatives such as the 40-day courses for primary teachers in the 90s and the MAST programme in the 2000s were not sustained for long enough to maximise the impact that they were having (Mathematics educator 7);*

> *Substantial funding is also required to allow classroom teachers to access high quality professional learning opportunities which are enquiry-based and demand a high level of criticality, as in the ACME reports (The Scottish Mathematical Council).*

4.7.4 Mathematics educators in the Call for Views and webinars expressed a strong call for government to mobilise an array of mathematics education, industry, academic specialists to develop key curriculum principles, but then allow phase- and subject-specific experts to develop those into documentation and professionals autonomy in implementation: such collaboration could also enhance teacher knowledge, especially of data handling and statistics. A new independent curriculum and assessment body would need to be established to oversee the co-development of e.g. a ten-yearly curriculum and assessment review, collaborating with publishers and awarding bodies to enact changes in a cyclical manner. Such activities could not operate stably and authoritatively as a government agency, although they would be funded by the state, so would need cross-party support.

4.7.5 The range of categories of respondents considered that valued goals required considerable investment in education, and in mathematics graduates; some argued to pay STEM qualified teachers more, so we get passionate teachers who care about young people's progress in subject-knowledgeable ways. Specialist users in particular embraced incentive funding for mathematics teachers, and enhanced funding, and entitlement, to ongoing mathematics-specific professional development. They also embraced improving the status of mathematics in society through 'incentives for STEM degrees; maths competitions, Olympiads, graduate level prizes…'.

4.7.6 A successful initiative of this magnitude was argued to draw on the range of interested communities' resources and active support, to encourage and support women and students from under-represented groups to engage with mathematics:

> *The Operational Research Society's programme "Operational Research in Education" works with schools to show how mathematical modelling and operational research approaches to show the wide applicability of modelling to a wide range of issues that school students can relate to (The Operational Research Society).*

Some mathematics users suggested the possibility of a national campaign at least comparable with 'Maths Year 2000' to celebrate mathematics and with attention to cultural relevance and the history of mathematics. It was felt this would ensure greater diversity in participation, as well as its active embrace in other disciplines.

4.7.7 There were repeated messages about the importance of engaging both girls and boys, and increasing diversity and equality of access to mathematical pathways. Many thought that more appropriate curricula, pedagogies and assessment could begin to address some issues around diversity, equity and inclusion in mathematical pathways. However, some also identified that these issues appear from much of the evidence to be stubbornly persistent, and deeply culturally-rooted. Several respondents, including in webinars, suggested influencing families was also critically important:

> *Parent/family workshops to increase enjoyment of and awareness of mathematics connected to the everyday (Teacher 11).*

Curriculum

Most responses in this section focused on the curriculum needs of those who would be more modest users of mathematics, with little input on how wider and deeper mathematical pathways might be developed to extend such learning.

4.7.8 There was often an at least implicit message, including from webinars, that much of the content scope needed universally is currently addressed by the end of primary school, though a more exploratory, connection-supporting, deeper, and more contextualised, often problem solving approach to that, resulting in a slower 'coverage', would give young people a more robust, and interesting, foundation. Additionally, respondents in webinars felt the primary curriculum should be enhanced with substantial realistic measurement and data handling work, supported by appropriate digital tools, and with cross-curricular and age-appropriate approaches:

> *Too formal too early is a big problem and leads to children building on sand: developmental trajectories are critically important… Cross-curricular work and use of digital tools for a range of purposes can easily be integrated into primary learning: children need to know why and how things work, and have a route to finding out if they forget things (Webinar);*

> *The mathematics in primary school being designed to enable students to understand mathematical structure and make connections between topics. Secondary mathematics to have a small amount of additional content but far more emphasis on applying, reasoning and problem solving more ak in to Core Maths (Teacher 30);*

> *The components outlined within the Pisa 2021 Mathematics Framework are welcomed. However, many of the skills assessed are not currently delivered within Scotland's curriculum (Scotland combined response).*

4.7.9 The importance of a curriculum that supports connection-making, across and beyond mathematics, and from an early stage, was widely argued. There was a small but significant, and approving, mention of 'Core Maths-type' curricula across several groups, with the variety of Core Maths specifications (currently available at level 3) being thought to represent well the balance of aspirations for mid-older teenagers' core mathematics education. One suggestion was that such courses could also be developed at levels 1 and 2, though perhaps with further-enhanced use of digital tools.

4.7.10 There was also widespread support for financial education being included from early in the mathematics curriculum: money habits are known to be formed by the age of 7, according to the Money Advice Service. Some thought common mathematics pathways should stop at 14, with others advocating a more 'nested' structure throughout. Many advocated elementary programming and computational thinking being integrated into the mathematics curriculum (*e.g. Geometric understanding is key to later development, and position-direction-movement coding via e.g. Logo or Scratch, is appropriate in the primary curriculum (Webinar)*), and mathematics should continue to feature as a fundamental part of interdisciplinary learning activities across the curriculum.

4.7.11 Students were widely said to need a balance between existing core skills and new skills in statistics and data literacy, across the mathematics curriculum; however, there was only one call for greater curriculum time devoted to mathematics: rather, new skills, content and emphases should replace those which are now less important, and secondary curricula build from primary school learning. However, there were also messages, sometimes implicit, about the overall school curriculum priorities adopted:

> *In primary schools, mathematics is often treated as the 'poor relation' with difficulties in working with numbers seen as less limiting to learners than difficulties with reading and writing…Furthermore, many primary teachers often find identifying opportunities for the application of literacy skills within cross-curricular projects easier, hence learners often miss out on opportunities to apply and practise skills acquired in 'maths time' in new and unfamiliar contexts. (Scottish Mathematical Council).*

4.7.12 Some tensions emerged between a few respondents (across several groups) calling for more mathematical rigour at every stage of the mathematics pipeline, and those arguing for greater breadth of mathematics study that includes considerable application and communication. Occasional opposing views included 'more software in schools; open book exams; learn to look up what you need' vs 'leave technology, increase rigour, proof, logical argument, arguing a solution, unseen problem solving':

> *Teach rigour; rules as axioms; each year teach the same topics but at a deeper level (Computing professional)*

4.7.13 The majority view, though, was to widen the mathematics curriculum, with data literacy developed across mathematics and many other subjects, and Core Maths, functional maths and/or data science type courses for all post-16, complementing other more focused mathematics courses for many; involve students in real-world analysis problems informed by industry and other users; increase applications in real world activities. However, a sizeable strand of responses thought 'pure maths is still important', to different extents for different young people: 'maths should not become 'demoted' to being viewed as a useful tool for careers in engineering and physics etc. without any inherent value'. Several respondents argued that pure mathematics should still be seen as a relevant and powerful focus of study for many young people, but essential for those intending to continue to make significant use of mathematics.

> *Future subject level reforms should aim at integrating data analysis into a wider range of subjects across the curriculum, including the humanities…(The British Academy);*
> *Solving interesting problems in geometry does more to brain than playing computer games (Academic mathematician 12).*

4.7.14 As a clear but influential outlier, Wolfram is promoting a computer-based curriculum[5] that had little buy-in from other respondents. However Wolfram also suggests that an effective mathematics curriculum would in large part be focused around meaningful problems, and that the aim should always be to choose problems which will:

- Be as realistic as possible to real problems students may actually face.
- Motivate them to enjoy mathematics and want to learn more.
- Build mathematical skills by introducing increasingly complex concepts rather than increasingly complex processes and procedures.
- Build an understanding of and competence in using an iterative four-step problem-solving methodology that has broad applicability.
- Give students as broad an experience as possible of today's mathematical tools (like machine learning).
- Develop complementary coding skills, at least some within mathematics education.
- Address a rather different set of mathematics outcomes than has been seen in traditional mathematics education.

There is much in this list that is consistent with emerging education research on the promotion of respondents' espoused mathematical competences, and with other proposals made in the Call for Views. Wolfram computer-based mathematics curriculum modules could, for example, form part of a computational-intense course appropriate to some students in upper secondary.

4.7.15 Another, outlier, proposal from a computer scientist was to 'create an online maths resource and allow learners to choose what they learn and when': it is not clear what age group this would be aimed at, and we now know rather more than we did last year about the affordances and constraints of independent online learning of mathematics, even for well-motivated 16-18 year olds with highly structured online curricula[6].

4.7.16 There was additionally, from webinars (though strictly out of scope), a call for an Early Years Framework more coherent with the evidence around young children's emergent mathematical capacities, including for work with shape and space, as well as number.

4.7.17 Further, from most groups there was also a low-key and persistent valuing of a curriculum which goes beyond the 'needs' on which the Call for Views was focused, so that young people come to see also the surprise, the satisfaction, the elegance, the 'big pictures' and enjoyment, as well as the frustrations and challenges to be derived from engagement with mathematics:

> *Emphasis on creativity, aesthetics, problem solving and the uniqueness of mathematics as the only universal language (Teacher manager 3).*

4.7.18 Finally, respondents working in education widely argued for stability once an expansive curriculum was in place, for removal of curriculum reforms from the political cycle, and instead, for systematic curriculum evolution that is negotiated strategically at a cross-party level, but is then systematically developed through collaboration across curriculum and education experts, in communication with stakeholders.

Pedagogy

4.7.20 Contributions around the pedagogies that might support an expansive curriculum focused around: a supportive, challenging, interactive and solution-focused classroom ethos; collaborative work; mathematical exploration and genuine problem-solving that would serve to deepen understanding of concepts and of mathematics as widely-applicable; and throughout with few exceptions, the pervasive use of digital

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[5] https://www.computerbasedmath.org/

[6,7] Golding, J., Redmond, B. & Grima, G. (2021). *Mathematics A Levels in a pandemic: Autumn 2020.* London: Pearson UK.

technologies to support such approaches, including for work with authentic data. More focused comments included:

> *The type of mathematical agency needed for citizens to solve the economic, social and environmental challenges they face individually and collectively can only be facilitated by collaborative, discursive, problem-solving pedagogies which embrace challenges, errors and misconceptions as learning opportunities…Students should be provided with opportunities to engage with rich mathematical tasks within a learning environment which focuses on sense-making rather than answer-getting (Mathematics educator 9);*

> *Online quantitative tools like UK government's Climate Calculator to give students and citizens a feel for maths (Computing professional 3);*

> *We need to focus on securing a deep and adaptable understanding of mathematics, to create resilience and flexibility to respond to the evolving needs of society and the economy (Home nation government department).*

However, some webinar participants noted that we do not yet know how much otherwise unnecessary 'doing' is needed to develop that deep conceptual grasp – or how to equip teachers to teach in such ways, at scale. Such issues clearly needs further research.

4.7.21 Rigorous development and use of mathematical language was often seen by those involved in education as a key tool to mathematical empowerment:

> *Mathematical language is sometimes seen as a barrier, but to children it's no more difficult than other words if it's introduced and used in a sense-making way, and rigorous and precise use of language actively supports mathematical thinking (Webinar).*

4.7.22 Mathematics educators also emphasised the importance of citizens feeling confident when working with mathematics. They widely argued this could be achieved by 'working collaboratively, posing questions, making conjectures, following original lines of inquiry, explaining and justifying solutions to others and considering alternative solutions: those should be an integral part of mathematics learning in schools';

> *Using a standard formula, or an inbuilt algorithm, to find something is not really mathematics. Making discoveries when solving unseen problems, however little these discoveries are, is very important (Academic mathematician 13).*

4.7.23 Some education professionals focused on key aspects of 'good pedagogy' known to be not yet widely practised:

> *Some learners disengage with mathematics from a young age as a result of a reinforced emphasis and dependence on memorisation of facts, rather than conceptual understanding. This could be addressed through the use of a 'concrete, pictorial, abstract' approach to learning and teaching….A shift to seeing manipulatives as a means of communication to making thinking visible requires to be encouraged and reinforced within the profession (Scottish combined response).*

4.7.24 Others pointed to the potential for high quality curriculum resources to support both specialist and non-specialist teachers of mathematics:

> *At NRICH we believe that the government should address the dearth of quality teaching materials currently available for developing collaborative problem-solving in our schools, as highlighted by*

Luckin et al (2017[7]) in their report 'Solved!'. Government should invest in high quality teaching materials which promote deep mathematical thinking so that students willingly and regularly engage with more challenging problems which engage and inspire them. These materials should be made available freely to schools, increasing their potential impact but also enabling all schools to access them equally (Mathematics educator 23);

Share resources rather better, especially via skilled teachers (BBC).

4.7.25 Most groups identified a need for pedagogies that support the developing positive dispositions for mathematical work, including persistence and resilience:

It has to be made clear that mathematics can be difficult and takes time to understand, that there's no expectation of immediate grasp, and that in fact one of the main points of mathematical activity is facing and overcoming that struggle. Students have to learn to concentrate and introspect in moments of confusion and non-understanding (Computing professional 9).

-and some, a need to support a greater diversity in mathematical roles through problems that are primarily situated in humanities and social science contexts as well in STEM areas.

4.7.26 In relation to the use of digital tools, it was noted that these have a potentially wide range of uses in the mathematics classroom, but for their mathematical potential focused on those used to support exploration of mathematical ideas and problems – for example, graphing software, elementary use of programming languages, data handling software. Digital manipulatives, and the constructive use of calculators to support grasp of place value and contextualised use of a wider range of numbers, were also mentioned in webinars.

4.7.27 Those more informed about current classroom practices and recent research pointed to the current very limited use of such digital tools in many UK classrooms, even compared with twenty years ago, to the complex and time-consuming demands on teachers in mediating deep student learning through the use of digital tools, and the persistent challenges in supporting teachers to use subject-specific software in the ways it was designed, and at scale. Examples given were the recent large-scale Cornerstone Mathematics[8] and Scratch[9] studies. Similar difficulties have been encountered in introducing digitally-supported curriculum requirements such as the study of a large data set in England's A Level Mathematics[10]. The result was that at present, too often such tools were used effectively only in 'digital enthusiasts'' classrooms.

4.7.28 It was felt that other uses of digital tools, such as for presentation, or access to information or videos that brought outside expertise into the classroom, were currently rather more commonly used. This theme is explored further in chapter 5, in the light of evidence emerging from the first pandemic year.

4.7.29 Some respondents noted in relation to pedagogies that even with computers and calculators, learners still need the ability to critically analyse output and question whether the 'answer' makes sense. That is dependent on the user having sound number sense, that is a deep 'understanding of numbers, their magnitude and how they are affected by operations'. Overall, it was felt that technology now potentially allowed learners a greater focus on conceptual understanding, and it was important that potential flourished,

[7] Luckin, R., Baines, E., Cukurova, M., &Holmes, W. (2017) Solved! Making the case for collaborative problem solving. Nesta solved-making-case-collaborative-problem-solving.pdf (nesta.org.uk)

[8] Cornerstone Maths | Institute of Education - UCL – University College London

[9] UCL ScratchMaths | Institute of Education - UCL – University College London

though there were some responses that urged caution in the use of digital tools, and pedagogies that address mathematics as a valuable activity in its own right:

> *Keep computers and IT at bay: these are tools that can enhance our abilities, but not replace them; being able to use computers is not a cure for not being able to think and reason..... Mathematics must be taught as a subject with enormous innate richness, beauty and interest, and not something that is valuable for its applications. Reducing Maths to a "useful science" quickly makes it dull. Mathematics should be studied and taught because it is interesting, taught as a game that is challenging and interesting to play, and then its power in helping us solve many problems and develop technologies comes almost as bonus (Academic mathematician 13).*

4.7.30 We note that the responses relevant to this section predominantly focus on ways to improve meaning-making, participation and mathematical functioning for critique and problem-solving: implicit in many of these is a reduction in the range of content addressed, in favour of deeper engagement with it. The curriculum implications, particularly for those needing a wider range of mathematical tools, are not clear, and there were a small number of counter-narratives suggesting that the development of mathematical rigour and deep, reflective fluency with mathematical ideas at a school level are not compatible with such approaches.

4.7.31 Further, webinar participants reported that mathematically-effective use of digital tools is highly demanding of teachers' subject, subject pedagogical, and technological pedagogical skillsets, underlining the need for extensive access to high quality professional development. Responses sometimes further argued that if such uses draw on beyond-mathematics contexts, then beyond-mathematics collaborations with colleagues, as are common in England's 'Core Maths' settings, become increasingly important.

Assessment

4.7.32 Responses to several questions, and across most groups, identified the need for (summative) assessments and/or accreditation to fully reflect curriculum priorities if those priorities were to be achieved – and that short timed written papers are ill-suited to assessing many aspects of the expansive goals suggested:

> *The ability to work things out, solve problems and puzzles should be valued, the ability to reason and present a logical argument should be valued – and that means accrediting it in some way (Teacher 30)*;
> *Designing an appropriate, agile, system of assessment and assessments which capture the right kinds of achievement, enabling students to demonstrate what they can do (MEI).*

Respondents from several groups commented that current assessments, though, often undermine such intentions, supporting 'content coverage' rather than 'digging deeper'. As might be expected, responses from those working in education were the most detailed and informed.

4.7.33 One respondent described one impact of mis-alignment with valued outcomes:

> *Since there is not much in mathematics A level assessment about real-world data use and problem solving, little about general critical thinking, and limited coverage of communication skills, we end up with a large number of people at the stage of university entry who either do not possess these skills and concepts, or have developed them out of mathematical contexts in other A levels. I can't see a good way to break down this cycle without changing post-16 examinations, possibly changing other things earlier in the school years, and going on to change the balance between pure and technical mathematics and the use of mathematics at university (Academic mathematician 20).*

4.7.34 There was therefore a fairly pervasive wish to end the 'high stakes' assessment model (though with a very small minority wanting to increase the rigour and mathematical demand of current assessments). A range of alternative suggestions included

- Two flexible mathematics 'GCSEs': one focused on numeracy including the interpretation of data, exponential graphs, number skills, space skills, calculator skills, reasoning, problem solving, use of technology; with the second more about the beauty of maths - algebra, trigonometry, circles, proof, quadratics etc... where both make 'good use of technology';
- A range of mathematics accreditations, at different levels, some focused on data literacy or 'Core Maths' or 'Maths with Computing', or….that might increase diversity and participation in mathematics-intense (including STEM) courses post-18;
- Flexible qualifications that combine examinations with portfolios and accreditation of accumulated experiences;
- 'Core numeracy accreditation to be taken at any age? Might need different forms at different ages, but then there would be a need to continue to use and build on that: should accreditations be nested?'
- A move towards a combination of take-when-you-are-ready computer-based tests for routine skills and short but equally weighted controlled problem-solving assessments marked by teachers or examiners (though others recognised a significant difficulty in assessing problem-solving reliably and validly on an individual and short-time basis);
- Qualifications constructed in such a way that people can create their own course to a specified level of achievement. Learning should be open to all, with formal institutions only having to solve the problem of certifying qualifications (from a computer scientist).

4.7.35 Some webinar participants identified a need to consider qualifications alongside assessment, wondering whether high stakes individual qualifications are needed in all curriculum areas, or all parts of mathematics education, or whether some could instead accredit experience and collaborative achievements. Timed written papers were fairly widely argued to still have their uses, and there is currently considerable investment in developing digital versions of such assessments (with some persistent doubts voiced about validity via such tools).

Teacher equipping

4.7.36 Although responses from most groups identified a need for 'more, better-qualified' teachers of mathematics, those close to education sometimes probed that more deeply, including in webinars. Some argued that a mathematics degree supports career-long further learning of new foci in mathematics, and new approaches to the subject, while also enabling a teacher to share mathematical 'horizons' with students. Others pointed to the evidence that effective teachers of mathematics teachers largely draw on deep knowledge of elementary mathematics and its pedagogy, to a somewhat higher level than they are teaching; one identified that this by itself would not necessarily be sufficent for teaching children with exceptional mathematics potential.

4.7.37 Many competences for teaching mathematics from a non-mathematics degree were claimed to be achievable through well-structured, sustained programmes which have a dual focus on subject content and subject pedagogy (for forward-looking purposes, imbued also with pervasive use of digital tools and the related pedagogy). Examples have included the MAST programme for primary teachers, and recent large-scale 'Mathematics enhancement courses' offered to those without a degree in a mathematics-intensive subject in England.

4.7.38 One policy organisation identified that we should learn from past reports on teacher recruitment, retention and career-long professional development needs, even before targeting a more aspirational curriculum, and also maintain funding for programmes shown to be effective, such as the MAST programme, while there is still a need.

4.7.39 A few respondents implicitly thought that a mathematics degree *fully* equips for effective teaching of mathematics, whereas others explicitly pointed to the need to complement that with range of knowledge of pedagogical preparation and other knowledge. There was, though, recognition that in international terms, the prior expectations and preparation of teachers of mathematics in England is currently very modest, as are the expectations on them, and opportunities, for subject-specific further professional development – and in no way comparable with that in the high performing jurisdictions whose mathematics education practice is widely admired, and targeted, in recent initiatives[11]:

> *In Ireland there was a push for all mathematics teachers to have some mathematical qualification. This was a requirement and there was a retraining programme. Until there is similar legislation in England, the problem of a shortage of mathematics teachers will persist…..There is a world-wide shortage of people with e.g. statistics skills so those with suitable skills are in high demand. Encouraging more young people into mathematics in the first place will help as well as making mathematics education a more enticing career option (SIG of Royal Statistical Society).*

4.7.40 There was one argument that a shortage of well qualified mathematics teachers can be overcome by building an online maths resource and allowing learners to choose what they learn and when; others warned against a simplistic approach:

> *(There is the) possibility of disruptors using technology to replace schools or teachers in an ill-considered way which reduces and/or damages access to effective maths education for all (MEI).*

Some respondents, especially those in education and in webinars, pointed to the need for teachers to mediate mathematical online learning in some way, for most young people much of the time. However, there was also in webinars a recognition of the role of digital technologies in bringing 'particular expertise' into the classroom, complementary to that of the teacher.

4.7.41 Mathematics educators commonly argued for a greater focus even for present needs, on the subject-specific mentoring of beginner teachers, including at primary, drawing on a high level of subject- and subject pedagogic specific knowledge, and career recognition of the importance of that role. One pointed out that in many secondary mathematics departments in England, there is no-one with even a mathematics-intensive, let alone a mathematics, degree, so such expertise does not necessarily exist within the school: there is a need in such circumstances to harness external expertise.

4.7.42 However, there was also recognition of the benefits of including teachers with wider backgrounds into mathematics education:

> *Education Policy Institute suggests that the challenge of recruiting and retaining teachers with appropriate qualifications in quantitative subjects will require regionally based solutions rather than a one size fits all approach….. These solutions could include maximising the value that can be gained from teachers who have quantitative skills developed through other disciplines, such as the quantitative social sciences, and supporting them through professional development to teach core*

[11] e.g. TALIS - The OECD Teaching and Learning International Survey - OECD

quantitative competences in the context of other subjects, thus also enabling students to understand how such skills can be applied to real-world problems (The British Academy).

4.7.43 Some responses cited ACME and other teacher-related reports (e.g. ACME, 2013[12]) showing that teacher retention and continuing education are intimately related to their opportunities for revitalising and empowering professional development, including through collaboration, and respondents mirrored that:

Engagement with the mathematical research community, especially mathematics education professionals, will be needed, and I believe we need more education professionals who examine statistics education in particular (Academic mathematician 9);
Teacher professional development as both expectation and entitlement; opportunities for teachers to work together to develop a curriculum (Mathematics educator 20).

In chapter 5 we briefly address digital pathways to such development.

In summary,

Key theme 6: Addressing these challenges would require unprecedented collaboration and investment

- There was frustration that a number of strongly-evidenced reports and policy initiatives focused on developing mathematics education, and teacher education, to meet even current mathematical and digital needs, have not borne significant fruit. Such shortcomings will have even greater impact on our individual and societal thriving if mathematical needs increase further, and it was thought impediments to full fruition need identification and addressing, if ambitious new visions are to be achieved.
- Re-shaping mathematics and digital education in the expansive ways envisaged would require unprecedented collaboration across the range of interested parties.
- It would need substantial, sustained investment in subject-specific and systematic teacher development that is career-long, and a cross-party political will to be seen to value mathematical competences.
- Only with such changes could we address the challenge of equitably equipping a diverse range of young people to choose to continue working with mathematics, at a variety of levels.

Key Theme 7: Critically, curricula, pedagogies and assessment would need coherent re-envisioning

- Mathematics curricula, pedagogies and assessment were thought to need radical and sustainable re-thinking in ways that are coherent with one another, and with the goals outlined. High quality resources also coherent with these would further support valid curriculum enactment. In this widely-embraced vision:
- Mathematics *curricula* would focus on mathematical concepts and key processes of mathematics: problem solving, reasoning and communication, as well as core skills, techniques and knowledge of elementary mathematics; they would address statistics and data literacies and contribute to developing digital literacies and computational thinking. /contd.

[12] ACME. 2013. *Empowering teachers: success for learners*. Advisory Committee for Mathematics Education [cited 14 December 2013. Available from http://www.acme-uk.org/media/14054/acmepdreport2013.pdf.

- Mathematics *pedagogies* would harness authentic examples, using digital technologies to focus on conceptual development and problem-solving; young people would be supported to develop rigorous reasoning and critical, collaborative and communication skills; pedagogies would be appropriate to the mathematical aspirations and maturity of learners, so that young people would both enjoy and appreciate mathematics as a cultural activity with satisfactions that transcend immediate needs.
- The main purpose of large-scale summative *assessments* would be to validate achievements, and their format in many cases would move away from timed written papers, towards more portfolio-based and other evidence that validated collaborative, problem-solving and often digitally-supported competences and possibly experiences.

4.8 What else should be considered in order to answer the two core questions?

4.8.1 A variety of additional evidence was offered, with key references within that evidence listed under 'References' below. Other additional points made included:

> *A lot of this work can be linked in with the IDSSP, International Data Science in Schools Project, where much of this is outlined for students in the final two years of school education (Academic mathematician 27).*

4.8.2 One policy-focused organisation was concerned that the project full considers the needs of vocational pathways at different levels, rather than focusing on purely the academic content and pathways, and another respondent that the government should

> *recognise, and fund interventions for, developmental dyscalculia, which has a prevalence and impact similar to that of dyslexia (Computer science educator 1).*

4.8.3 The British Computing Society, while saying that mathematics and computing education should be kept as separate curriculum strands, argued for an expanded project scope:

> *We urge the programme to treat computing education as part of its scope.... Through computing some of the most abstract mathematical ideas have had a concrete impact on the way the world works. Ubiquitously available computing devices, big data, the cloud, and all their implications, is the big new thing on the maths education horizon, compared to say 30 years ago.*

4.8.4 And several respondents commented on further developments of the Mathematical Futures programme:

> *Scenario planning definitely, needed as a base level for employment, citizenship and consumers (BBC);*
> *Look at the big 5 tech companies, profile which of their jobs are growing fastest, and work out whether our curriculum will equip our next generation to be chosen to fill these job gaps when competing with others from around the world (increasing data scientists, data engineers, cloud computer architects, roboticists, ethicists etc.) - the countries that can provide these the most effectively will have the lowest unemployment rates (Specialist user 3);*

> *Communicating and influencing: When trying to communicate and influence key stakeholders, the programme should be precise about what it is wanting to achieve, who it is trying to influence and how. For policymakers, think about what is the story around what we are trying to do? Why now? Who are we trying to influence? What are we trying to get them to think? Are you trying to get it widespread or narrowcast? Then, what are the hooks? (BBC).*

5. Discussion

Respondents, including in webinars, were almost always knowledgeable either about mathematics, its education, or the implications of mathematics education for the workforce, and argued that the re-envisioning of mathematics education and of its workforce, to embrace both the increasing availability and power of digital tools, and the consequent proliferation of available data, for both personal and societal mathematical purposes. The report does not, though, substantially evidence the views of 'the man in the street' in relation to the use of, or education for, mathematics, and it is important that the political implications of a response on the scale suggested has multilateral support.

Call for Views respondents largely focused on competences they wanted to see emerging from secondary education, rather than what underpinned that at a primary, or early years stage, though webinar participants provided some views on that, largely supporting a re-focus on confident grasp of process and concepts within a smaller, though data- and digitally-enriched curriculum – but also identifying how hard it is for non-specialist teachers to achieve that. Some groups, notably academic mathematicians, offered few specifics about the education needed to underpin their own work.

While respondents pointed to a need for unprecedented commitment, investment and collaboration in achieving valued goals, only occasionally was there any recognition of the current sustained and significant investment being made in Maths Hubs, and then from those close to the programme. However, The OECD TALIS studies[13] evidence the low level of initial teacher education investment made in England, and the comparatively meagre mathematical, and mathematics pedagogical preparation typically received by teachers of mathematics in England.

The medium-level vision that has broad agreement is in many ways coherent with high-profile initiatives such as the OECD 'Curriculum Futures'[14] work, and the direction of travel in some high-performing jurisdictions with more systematic approaches to curriculum development than some parts of the UK. It is important to note it also has some synergies with the most recent curriculum initiatives in each of the UK home nations, though in different ways, and indeed, one frustration expressed is that well-supported re-emphases on, say, problem solving, reasoning and communication in the English 2014 national curriculum are still to bear significant fruit: the reasons identified for that are critical to the success of any future initiative.

However, a number of respondents urged caution in not moving too far in the less discipline-aware direction of Young & Muller's (2010[15]) 'Future2' scenario. Further implications of the proposals, including solutions, made, including how much we *do not yet know* about what types, extent and depth of factual and procedural knowledge are needed to underpin a more conceptual and strategically focused curriculum, at different developmental stages.

One significant wider-community critique of both the OECD 'Curriculum 2030' proposals for mathematics, and Wolfram's computer-based curriculum proposals, is that until young people have conceptual tools, including concepts of the purposes, structures, affordances and constraints of mathematical processes, they are not in a position to select and apply those tools as appropriate to the situation in focus, even if via a computer. We need to know more about effective ways to build such conceptual knowledge if the long-term goal does not include carrying out all procedures by hand.

[13] TALIS - The OECD Teaching and Learning International Survey - OECD

[14] OECD Future of Education and Skills 2030 - OECD Future of Education and Skills 2030

[15] Young, M. & Muller, J. (2010). Three educational scenarios for the future: lessons from the sociology of knowledge, *European Journal of Education 45*(1), 11–27 DOI: 10.1111/j.1465-3435.2009.01413.x

However, existing and emerging curricula in related fields could productively be analysed for their applicability to UK contexts: the IDDPS digital literacy curriculum, for example, could fairly easily be incorporated into 14-19 education, the new Welsh curriculum has ideas to offer in relation to cross-curricular work, the Scottish curriculum for increasing the participation of girls post-16, and current English Core Maths-similar specifications could be developed and adapted for level 1 and 2 purposes, given a more flexible approach to framing curriculum.

The use of digital tools for education purposes has of course enjoyed massive scrutiny during the last year's pandemic, though in mathematics education that has largely been for communication and presentation, rather than subject-specific, purposes. Emerging evidence[16] suggests that teachers have made significant progress in learning to harness technologies for some purposes, for example blending available videos and pre-structured teaching sequences with their own input, so bringing 'external expertise' into the classroom. The use of graphing technologies, or spreadsheets for exploration of sets of data, or dynamic geometries, in the mathematics 'classroom' has to date remained a minority approach.

The pandemic has also exposed considerable inequities in access to digital technologies, and to the home-based space and support needed to mediate effective remote working[16]. Especially for younger children, there is evidence the pandemic year saw marginalisation of those areas of the curriculum teachers perceived to be more difficult to access remotely, such as reasoning and problem solving, or deep conceptual grasp of fractions, and in mathematics that often meant focusing on the easier-to-learn, routine, aspects of the subject[17]. The related learning costs are hard to quantify, since these are often also the aspects of mathematical functioning it is also harder to assess. There are clearly lessons to be learnt before we embrace digital technologies as, for example, a panacea to shortages of specialist knowledge.

However, there is also some evidence that teachers in the UK have embraced remote professional development to an unprecedented degree during the pandemic, and that approach appears effective for many aspects of teacher re-equipping, though there are still some questions about how to develop face-to-face pedagogies effectively when working online[18]. Such pathways, while limiting incidental social interchange of experience and professional wisdom, allow for flexible and time-efficient participation, and can sometimes also prove cost-effective.

International assessments have offered some encouragement in the area of attitudes towards mathematics: for example, recent PISA22 field trials[19] in England, Wales and Northern Ireland, and TIMSS19 findings[20] for England show an increasing valuing of mathematics from previous cycles, by 15-year olds and year 9 students respectively, but a stubborn association of confidence to function mathematically with gender, and with socio-economic background. The former also suggest that girls are typically interested in different uses of

[16] Education Policy Institute & Renaissance Learning (2021). *Understanding progress in the 2020/21 academic year – Interim findings*.https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/962330/Learning_Loss_Report_1A_-_FINAL.pdf

[17] Golding, J. & Grima, G. (2020). A pandemic summer: Impact on teaching and learning for mastery in Power Maths primary schools. http://www.bsrlm.org.uk/publications/proceedings-of-day-conference/ip40-3/

[18] Golding, J. and Bretscher, N. (2018) Developing pedagogies for a synchronous online course on teaching pre-university mathematics *Teaching Mathematics and its Applications* 37/2, 98-112 https://doi.org/10.1093/teamat/hry010

[19] Hill, M., Custodio, I., Golding, J. & Grima G. (2021). *A study of gender, self-perception, and mathematics: The 2020 England, Wales, and Northern Ireland PISA Field Trial.* Presented at: 2021 AEA-Europe annual conference, 3-6 November 2021.

[20] Richardson, M., Isaacs, T., Barnes, I., Swensson, C., Wilkinson, D., Golding, J. (2020). Trends in International Mathematics and Science Study (TIMSS) 2019: National report for England (publishing.service.gov.uk)

technology than boys (e.g. for social purposes rather than gaming), and boys, and those from more affluent background, are typically more confident of success in digital task such as systems analysis, programming, debugging, …. that underpin many occupational opportunities. There seems still to be some way to go in achieving equitable participation in mathematical and digital technology activities.

In relation to use of digital tools, there was some mixed use of terminology such as computing, computational skills, I(C)T, computer studies, digital skills …. It would be helpful for forward-looking discussion if the use of such terms could be agreed across the communities close to their key uses.

Finally, respondents pointed to the desirability of a mathematics education that provides for more than near-future personal and economic *needs*: the study of History, or of Art, within the curriculum would not eb supported if that were the only criterion of worth. They argued for mathematical activity to be understood as fundamental to all known human cultures, satisfying and intriguing in its own right, as well as personally and societally empowering. Such arguments of course return to the imperative for a mathematically deeply-equipped workforce also confident to draw on other sources, including digital ones, to educate young people for a mathematically confident future.

6. Conclusions

The response to this Call for Views has opened a box of challenges, exciting ideas, informed opinions and creative thinking. We have tried in this report to do justice to the myriad views, including those presented at three highly productive validating webinars, involving nine focus groups, in the last week in April 2021. Chapter 4 remains close to the data; in chapter 5 we were able to contextualise that somewhat in wider debates and evidence. Analysis, interpretation and contextualisation of this sort of data is necessarily informed by our own professional backgrounds in the mathematics and mathematics education communities.

Call for Views respondents presented a compelling case that current mathematical and digital competences in the UK are under-developed for our societal and individual thriving. It was argued that the recent pandemic has exposed the 'new normal' where mathematical approaches to ubiquitous data are seen to underlie our grasp of, and possible solutions to, a range of society routine needs and challenges. A sustained and collaborative investment in developing an expansive mathematics curriculum system to address the underlying demand for enhanced mathematical capacity, as part of a broad and balanced curriculum, is therefore required. Such a mathematics curriculum was very widely argued to need pervasive harnessing of the potential of digital tools for mathematical use, and to be supported by a teaching force mathematically well-equipped for such purposes.

Views suggest that any development of mathematics education should provide for the needs of at least three key (overlapping and fuzzy) sections of the population:

> *1. Mathematically functional citizens, who need a confident grasp of core numeracy, financial, data and statistical literacies, and use of digital technologies for those purposes;*
>
> *2. Those who in their occupation have more-than basic need for use of such mathematical functioning, working in traditionally non-quantitative jobs/professions, but now requiring increased skills: e.g., lawyers, journalists, civil servants, politicians, health, administrators, all teachers, vocational professions;*
>
> *3. 'Specialist' users of mathematics - analysts, engineers, financial professionals, many computing professionals, scientists, social scientists, of which one line is academic and professional mathematicians.*

Responses did not support a sub-division of the first group, although there is a range of mathematical needs within each group. They argued for universal education for robust basic numeracy (including geometric and very basic algebraic literacies), and for financial, data and statistics literacies, with most supporting development through a focus on conceptual understanding, authentic problem solving and mathematical communication and reasoning, via collaborative work and engagement beyond mathematics. Some education pathways would then build such competences further through wider and deeper mathematical ideas and applications, with education for the last group including an introduction to calculus, again focused on concepts as well as basic techniques. Details of the related valued curriculum were not yet well-specified, especially for the second and third groups.

Respondents felt it is important that choices of pathways within mathematics education are designed so as to allow for equitable and timely choices, and that those pathways should be flexible; also that they support productive mathematical dispositions, so that all young people come to identify themselves as effective users of appropriate mathematics.

But it was also argued that education should be designed more widely than such a summary would suggest: it is also important that mathematics education embraces an understanding of mathematics as a key cultural tool and source of delight, wonder, and satisfaction, and that it communicates, in inclusive ways, a grasp of its value to society and to individuals, including through digitally-enhanced experiences of its enriching and empowering potential. Only so would the range of our young people come to fully participate in mathematical ways of knowing the world.

We were surprised at the degree of consensus of views across respondent groups, though responses were very often at a high and 'in principle' level: development of proposals in greater detail might well expose further contestation. The overwhelming majority of variation, by group or individual, was in depth and focus, rather than representing a fundamental disagreement, and opposing and outlier views were unusual, as represented above. Rather more common were issues around balance - between mathematics conceptual knowledge development and experience with applications of mathematics; between development of 'by hand' skills to support concepts, and the use of digital tools which would also do that; and the degree to which data and computational skills should be developed within the mathematics curriculum. The place for computing education was also somewhat contested, although almost all respondents felt mathematics education (including harnessing of digital tools, elementary programming, and algorithmic and computational thinking all for mathematical purposes) should be retained as a separate curriculum strand from computing education, other than in deliberately cross-curricular projects. Such variation should not overshadow the wide support for a more expansive mathematics education as above.

The range of responses also exposed some areas that we suggest would merit further exploration in pursuit of the goals of the MF Programme. Such areas include:

- Routes to greater diversity, equity and inclusion in mathematical participation and employment;
- Paths to embedding previous, as yet unrealized but still current policy reports and initiatives: we need to understand why many of those have not come to fruition, if future, more ambitious developments are to succeed;
- Knowledge of how to re-focus curricula on conceptual grasp and wider mathematical competencies without jeopardising the core knowledge frameworks that underpin conceptual structures
- Knowledge of ways to develop beginner and experienced teachers so as to teach such expansive mathematics curricula in inclusive ways and so that the range of young people experience appropriate and authentic success;
- Clarifying the vocabulary used around computing, computational thinking, IT, digital skills – and around data science, so as to better support debates around their appropriate relationships with mathematics education.

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Further reading suggested by respondents

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Bynner, J. and Parsons, S. (1997). *Does Numeracy Matter?* The Basic Skills Agency. http://nrdc.org.uk/wp-content/uploads/2005/01/Does-numeracy-matter-more.pdf

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Grillenberger, A. & Romeike, P. (2019) Developing a Theoretically Founded Data Literacy Competency Model In *Proceedings of the 13th Workshop in Primary and Secondary Computing Education*, ACM, 2018.

Parsons, S. & Bynner, J. (2005). Does Numeracy Matter? London: The Basic Skills Agency numeracyv5.QXD (nrdc.org.uk)

For the mathematics needed by Fine Arts students see https://files.smartsurvey.io/2/1/1U0WV57I/151347425_12278900_1566335.pdf

On financial literacy and the curriculum imperative: https://files.smartsurvey.io/2/1/853HU16X/154921333_12278900_1614479.docx

Two research projects: https://wun.ac.uk/wun/research/view/innovating-the-mathematics-curriculum and https://gtr.ukri.org/projects?ref=EP%2FT003545%2F1 http://www.mathscraftnz.org/ ;

On developmental dyscalculia: https://files.smartsurvey.io/2/1/FLGIM4GF/151015336_12278900_1561831.pdf

On working with parents:
www.learningwithparents.com; https://www.earlyyearsmatters.co.uk/.../working-in-partnership

The British Academy SHAPE cross-curricular work: https://www.thebritishacademy.ac.uk/publishing/journal-british-academy/8/social-sciences-humanities-and-arts-shape-future-peoples-economies-environments/

Potential curriculum development resources referred to

- International Data Science in Schools Project http://www.idssp.org/
- OECD PISA21 (now PISA22) Mathematics Framework https://pisa2021-maths.oecd.org/
- https://www.letsthink.org.uk
- https://www.collaborative-lesson-research.uk/
- Wolfram's Computer-based maths curriculum https://www.computerbasedmath.org/

Appendix 1: The Questionnaire

A. The nature of mathematics in employment, society and citizenship

In the Mathematical Futures programme, the term ‘mathematics’ is used inclusively, covering a variety of ways of thinking, reasoning and solving mathematical problems which touch on many aspects of everyday life, work and study (in mathematics as well as in other subjects/disciplines). It includes quantitative skills and other activities of a mathematical nature such as those associated with numeracy, statistics, computing, and data analysis. Mathematical competences cover the way mathematical concepts, skills and understanding as well as attitudes are brought together when applying mathematics to solve problems

1. Which mathematical competences are most useful to you (or your employees) and why?
2. What are the most useful mathematical competences that citizens need now and why?
3. Do you think the nature of mathematics, and the role it plays, have changed over the past twenty years? If so, how?
4. Thinking about the needs of citizens, how should mathematics enable the next generation to participate in society?

B. The future of mathematics in education

This section relates to the programme's second core question: how should education systems develop these mathematical competences? Mathematical competences cover the way mathematical concepts, skills and understanding as well as attitudes are brought together when applying mathematics to solve problems.

1. What should be the main goals of mathematics education, and why?
2. What do you expect to be the challenges facing mathematics education in the next twenty years?
3. How could the challenges you have set out in your response to the previous two questions be addressed in practice?

C. What else should the Mathematical Futures programme consider?

The two core questions of the Mathematical Futures Programme are: • What mathematical competences will be needed for society to thrive in the future? • How should education systems develop these mathematical competences?

1. What else should the programme consider in order to answer the two core questions?

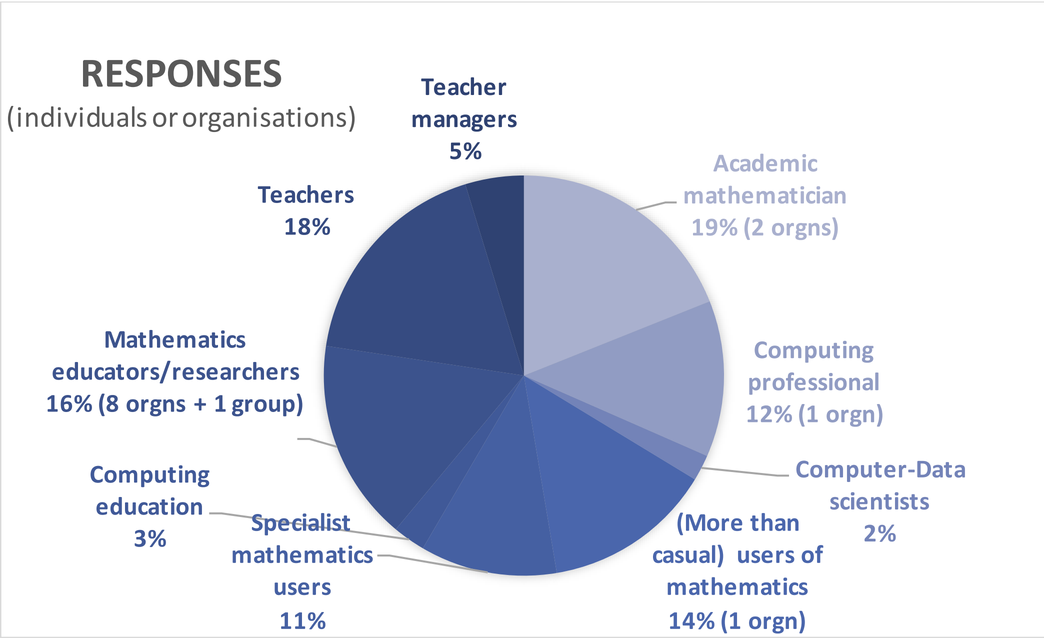
If you have any supporting evidence you would like to share, please attach it to your response.

- Would you be willing for us to contact you about your response to this call for views if required?
- Would you be happy for us to contact you in future about the Mathematical Futures programme?
- Please add any further comment

Appendix 2: Quantitative Data

Data from the core Call for Views were presented as open responses to 8 questions. The length, style and quality of responses varied considerably. Some respondents identified one or two key competences or challenges and gave detailed answers elaborating and justifying their answers, while others provided a list of high-level points without any detail. Some respondents gave a detailed thoughtful response but across the topic rather than linked to a particular question. As described elsewhere, the methodology set out to draw out themes, initially by respondent category and key principles and hence provide fundamental qualitative data that could then be compared across groups. Some key limitations and related decisions are noted in chapter 2, but those are detailed further here.

As noted, the online questionnaire resulted in 191 usable responses. These were systematically and iteratively organised into (inexact and porous) core category of respondent, to highlight synergies in responses by such categories. Data synthesised for this report were based on questionnaire responses as follows:



The response to each question was read and key points from each respondent open coded. For quantitative analysis supplementary to the overarching qualitative approach, these key points were summarised across a group of respondents so to identify a set of 5 categories for each question. We then coded the responses *according to whether they included (any number of) reference(s) to each of these categories: the diagrams show the proportion of responses making such reference, at any scale. This, of course, differs from a 'mention' count favoured by much qualitative software, which would count 5 references within a single response as 5 mentions.* Ours was an iterative process refining the categories as we analysed more responses, revising and reviewing as we progressed through the 191 respondents. The categories for each question are listed below.

Limitations of quantitative data: Our aim in quantifying the data was to identify key emerging themes and principles in order to further inform the qualitative data. The quantitative data on their own, for reasons outlined in chapter 2, are not robust and cannot be used to make claims.

We did not quantify the answers to question 1 (Which mathematical competences are most useful to you (or your employees) and why?) as these answers were as varied as expected when respondents range from early years teachers to a pharmacist to a professor of mathematics. We also did not quantify the answers to Question 3 (Do you think the nature of mathematics, and the role it plays, have changed over the past twenty years? If so, how?) as the respondents interpreted this question in many different ways and the varied answers were difficult to code in a consistent way.

The figures 2 to 7 illustrate the data for questions 2, 4 – 8.

The iteratively-devised categories that emerged from the analysis of the data for Questions 2, 4 and 5 are as follows, and encompass very nearly all data received. Many respondents provided answers relevant to more than one category.

- <u>Mathematical and financial literacy</u> including: skills for work and life; ability to estimate and 'is the answer correct'; appreciate size of number (£5million large or small?)
- <u>Knowledge and skills in statistics, probability and data analysis</u> including: interpret and question results; gain an understanding of uncertainty and risk; gain familiarity with mathematics software.
- <u>Cross curricular links and Programming / coding including:</u> applications of mathematics in other subjects; computational mathematics; higher level of mathematics for those who need.
- <u>Mathematical thinking, problem solving and reasoning skills</u>: including for real world problems as well as using digital technology.
- <u>Role and profile of mathematics</u> including: make maths accessible to all, increase diversity, improve attitude and confidence.

The data for questions 6, 7 and 8 were, similarly, sorted according to the following categories:

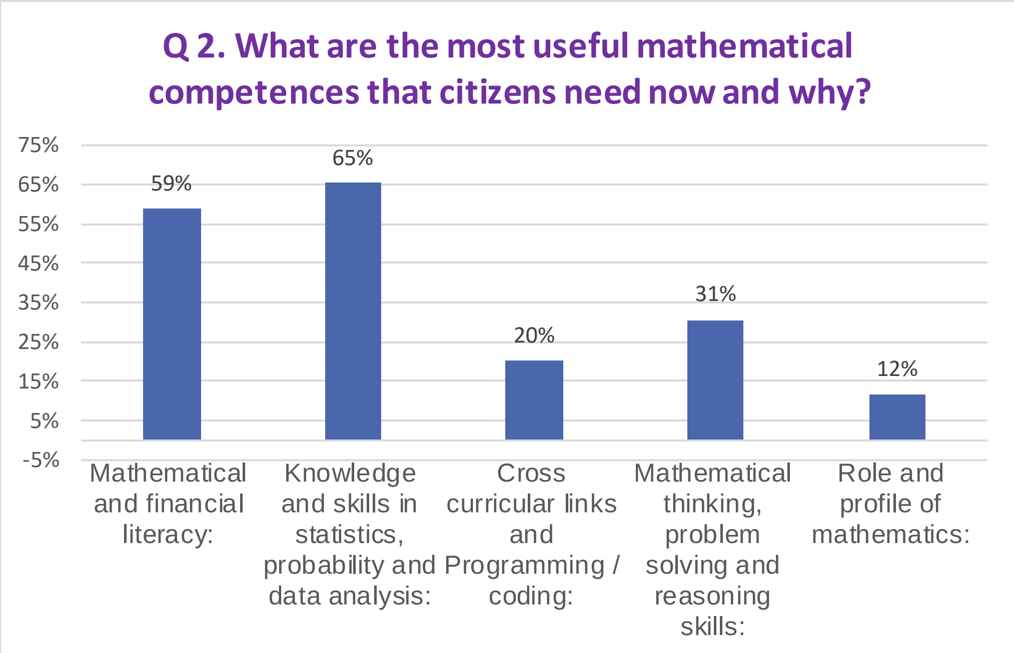
- <u>Investment in mathematics education</u> including: shortage of teachers with mathematics qualifications; need for Teacher professional development; better access and use of digital technology to support teaching and learning; research.
- <u>An appropriate curriculum and assessment model for the 21C</u> including: who is responsible; what is the content
- <u>Cross curricular links and Programming / coding</u> including: applications of mathematics in other subjects; computational mathematics; higher level of mathematics for those who need it.
- <u>Mathematical thinking, problem solving and reasoning skills</u>: including for real world problems as well as using digital technology.
- <u>Role and profile of mathematics</u>: make maths accessible to all, increase diversity, improve attitude and confidence; end fear of mathematics.

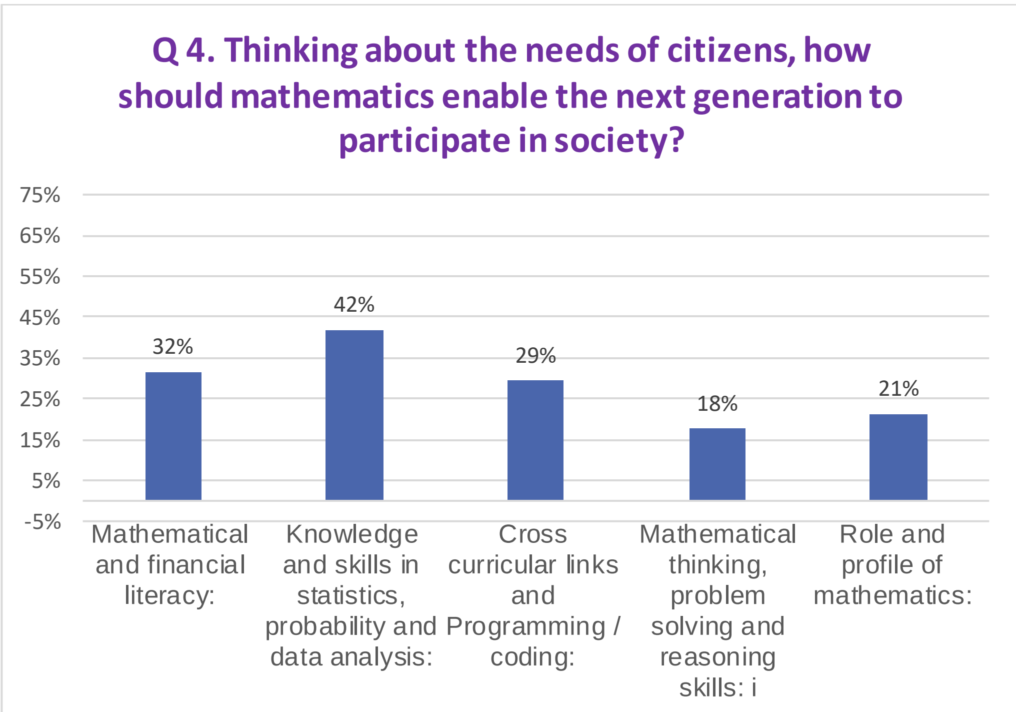
The weight attributed by respondents to each category varies across responses to questions 2, 4 and 5. For Question 2, a majority of respondents saw the most important competences were linked with citizens requiring basic mathematical and financial literacy, and skills and understanding around statistics, probability and data analysis. A number of respondents gave more detailed reasons these competences were important. These reasons are brought out in the main report.

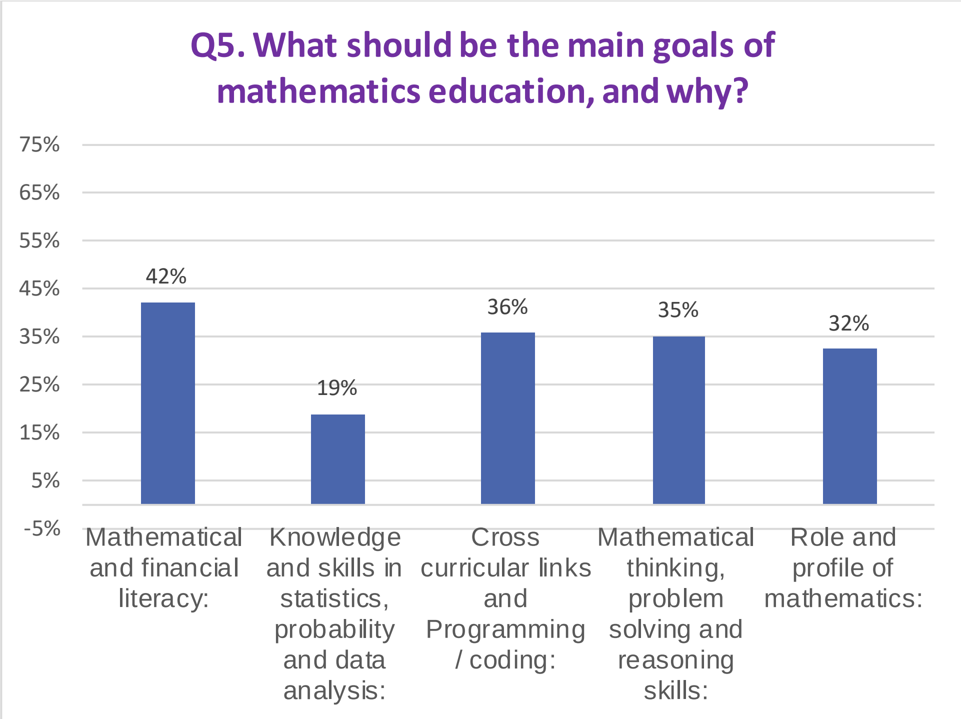
It is valuable to recognise that for many, basic numeracy must include a sense of order or magnitude of a number; grasp of 'is this answer correct?' (does it meet the parameters of the question?) and 'is that an appropriate number for an answer in this context?' The three remaining categories show less importance in question 2 but gain in importance in question 4 and the balance starts to even out in question 5. There appears to be a discrepancy in that category 2 (statistics, probability and data analysis) moves from 65% of respondents in question 2 to 19% in question 5. There is no evidence that respondents consider that an understanding of statistics loses importance but rather, that they absorb it in categories 3 and 4. Similarly, the high profile of that category initially might simply reflect an understanding that at present, related ideas have only a poor representation in current school curricula.

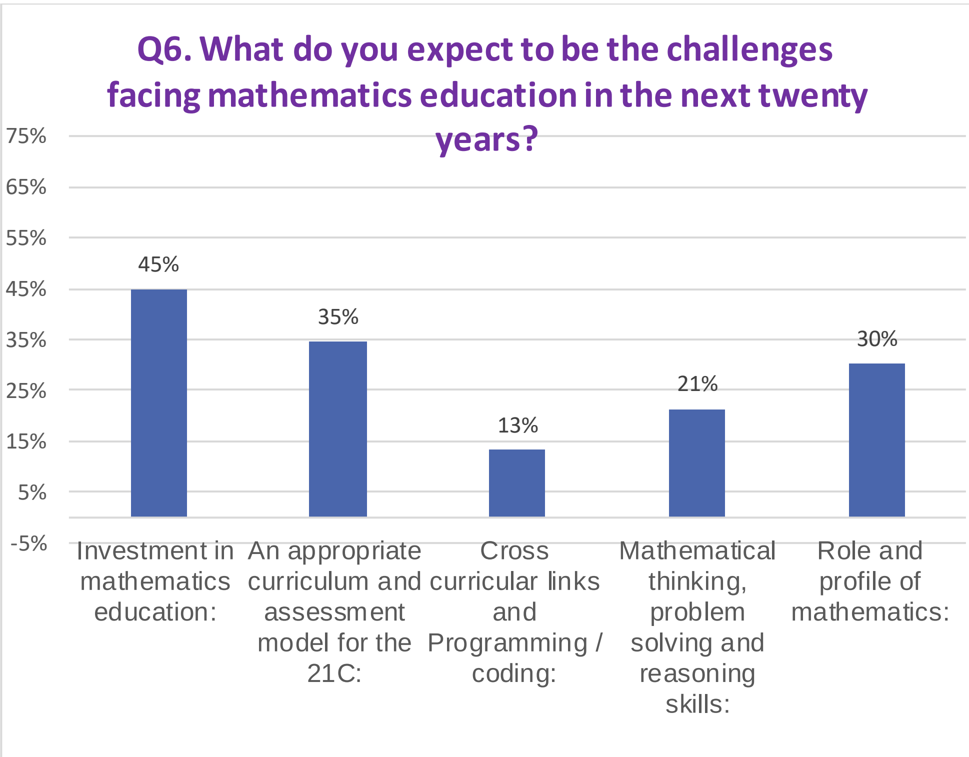
Questions 6 and 7 are about the challenges for mathematics education and here the themes change. The most prevalent challenges reported are focused on investment in education – shortage of well-equipped mathematics teachers, the need for career-long high quality professional development, including for teachers to gain confidence with using digital technology to explore big data, to teach coding and programming and to gain experience with the applications of mathematics across other subjects. The second priority area is the need for a curriculum and assessment model that responds to the needs of the 21st century.

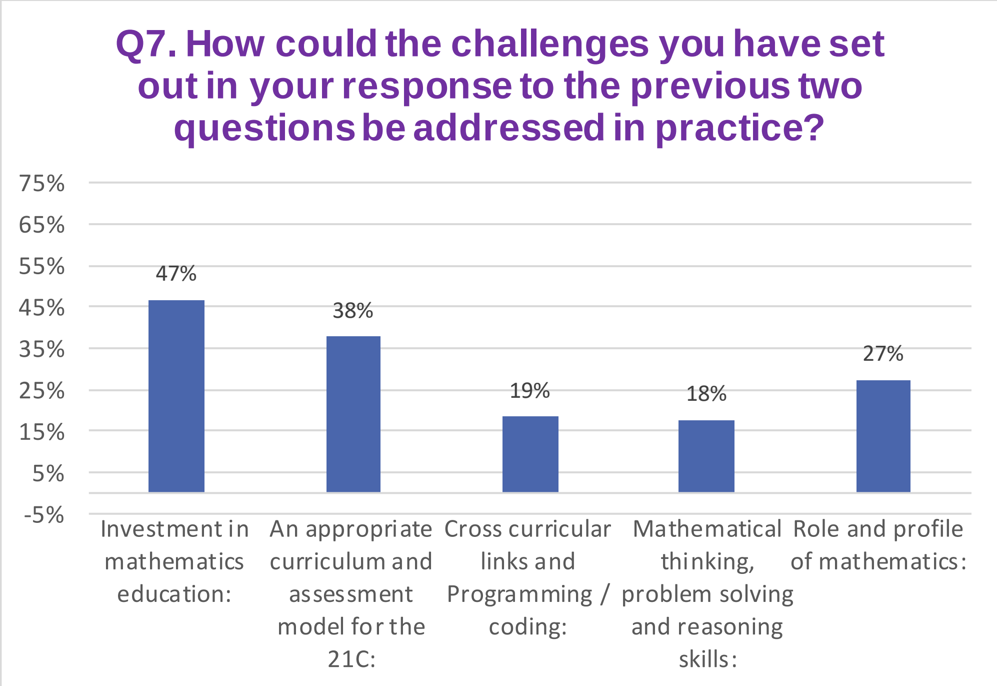
Note that across diagrams, the proportion giving any one category of response is not high, highlighting that respondents generally prioritised their responses, selecting messages that were key from their perspective. Across all questions there is a strong, even if sometimes small, response to raise the profile and public perception of mathematics. Without this it will still be acceptable for parents and children to say 'I cannot do maths'.

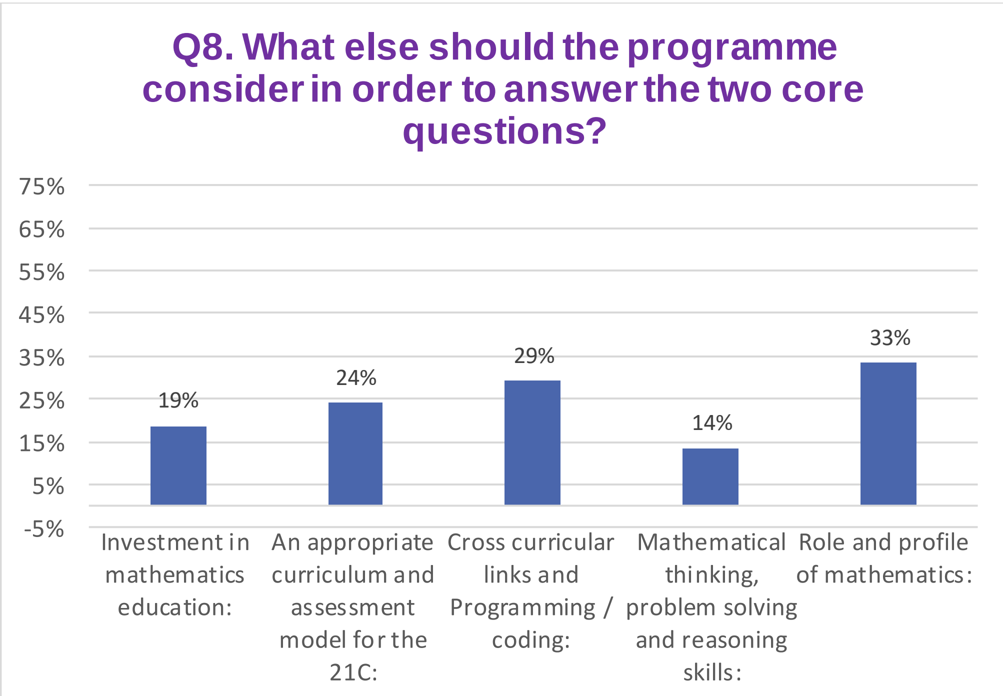





Q5. What should be the main goals of mathematics education, and why?
75%
65%
55%
45%
35%
25%
15%
5%
-5%
42%
19%
36%
35%
32%
Mathematical and financial literacy:
Knowledge and skills in statistics, probability and data analysis:
Cross curricular links and Programming / coding:
Mathematical thinking, problem solving and reasoning skills:
Role and profile of mathematics:


Q6. What do you expect to be the challenges facing mathematics education in the next twenty years?
75%
65%
55%
45%
35%
25%
15%
5%
-5%
45%
35%
13%
21%
30%
Investment in mathematics education:
An appropriate curriculum and assessment model for the 21C:
Cross curricular links and Programming / coding:
Mathematical thinking, problem solving and reasoning skills:
Role and profile of mathematics:


Q7. How could the challenges you have set out in your response to the previous two questions be addressed in practice?
75%
65%
55%
45%
35%
25%
15%
5%
-5%
47%
38%
19%
18%
27%
Investment in mathematics education:
An appropriate curriculum and assessment model for the 21C:
Cross curricular links and Programming / coding:
Mathematical thinking, problem solving and reasoning skills:
Role and profile of mathematics:


Q8. What else should the programme consider in order to answer the two core questions?
75%
65%
55%
45%
35%
25%
15%
5%
-5%
19%
24%
29%
14%
33%
Investment in mathematics education:
An appropriate curriculum and assessment model for the 21C:
Cross curricular links and Programming / coding:
Mathematical thinking, problem solving and reasoning skills:
Role and profile of mathematics: