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Ref:

Mathematics Education Pre-19

Summary and Key Recommendations

This paper, endorsed by Council, was produced by an *ad hoc* group of the Society's Education Committee. It considers current issues and concerns within mathematics education and makes recommendations on curriculum and assessment issues as follows:

- With the development of the National Curriculum the amount of time generally made available for mathematics has come under pressure and in many schools is diminishing. The Society is concerned by this trend and recommends that time allocated to the teaching of mathematics in Key Stages 3 and 4 is carefully monitored.
- The criteria for school performance tables, with their emphasis on the proportion of students achieving grades A*-C, may distort the tier which schools enter their pupils for at GCSE level mathematics: schools may take the safe option and enter more pupils for intermediate level GCSEs in the hope that a higher proportion will obtain grade C. This Society recommends that the effect of performance tables on GCSE tier choices be monitored carefully and that research be undertaken to establish the extent of their effect.
- A major concern of the Society is the relative difficulty of mathematics and science at A level compared to other subjects. The Society supports the recommendation of the Dearing Review of 16-19 education that this issue should be investigated as soon as possible.
- The Society notes the difficulty of recruitment to initial teacher training in mathematics and science. It is clear that there is a lack of highly qualified teachers entering the profession. The Society considers this a matter of the utmost urgency and recommends that it is tackled at national level.
- The role played by local authority specialist advisors in mathematics is considerably diminished in most authorities. The '20-day' in-service training (INSET) courses have been extremely valuable, yet DfEE funding for these has been withdrawn. The Society stresses that investment in the teaching profession is central to improving the quality of teaching and learning.
- The Society believes that, in order to address issues of national importance within education, more robust mechanisms must be put in place to allow for full debate amongst interested parties. The Society therefore reiterates its

support for a permanent advisory committee for mathematics within the recently created Qualifications and Curriculum Agency (QCA). Similarly, effective mechanisms must be established to ensure that an overview of non-curriculum issues such as teacher recruitment and training and in-service support is achieved. These areas currently cross departmental borders and advisory body responsibilities. The Society strongly believes that greater coherence is required in tackling these problems.

- The Society reiterates the conclusions of its 1997 report, '*Teaching and Learning Algebra Pre-19*', and particularly the need for an earlier introduction to algebraic material in Primary and Secondary schools.

Introduction

In recent years there has been an active debate amongst university academics, teachers, educationalists and others on the topic of mathematics education, particularly with regard to the preparation of students for higher education studies with a high mathematical content. Often this debate has been of high profile, commanding much attention from the media. A number of reports have been published, from both independent bodies and Government sponsored agencies, and a selection of these are listed at Appendix A. It was partly in response to the report, '*Tackling the Mathematics Problem*' (1995), that the Royal Society's Education Committee established an *ad hoc* group to consider in more detail the Society's response to this debate. Following the group's report to the Education Committee, the Society agreed to issue a statement on mathematics education pre-19, building on the conclusions of the *ad hoc* group and outlining issues and concerns within this vital area of education.

Concurrently with the work of the *ad hoc* group, the Royal Society supported, jointly with the Joint Mathematical Council (JMC), a report on algebra education entitled, '*Teaching and Learning Algebra Pre-19*'. Many of this paper's recommendations regarding algebra have been drawn from that report.

The Society acknowledges that there are problems to tackle within mathematics education. These relate especially to the most able students, but the problems are no less serious for that reason. There have been improvements, particularly for the education of the less able students, but these may have had unforeseen effects for other student groups.

Concerns have been raised by many academic teaching staff and admission tutors in the higher education sector with regard to the mathematical ability of those students entering university courses with a high mathematical content. The following aspects are most often highlighted:

- that many students lack essential mathematical facility, resulting in an inability to undertake algebraic and numerical calculations with fluency and accuracy;
- that there has been a decline in the standard of analytical ability, especially when students are faced with simple problems requiring more than one step to solution;
- that the perception of the nature of mathematics has changed, particularly with regard to the essential place of precision and proof;

- that the content of Advanced GNVOs in science and engineering, particularly in the mandatory units, does not contain sufficient mathematics to support further studies in these subjects in higher education.

These concerns have also been accompanied by increasing difficulties in attracting students to the study of mathematics in 16-19 education and in higher education. The proportion of the A level cohort who opt for mathematics has shown a decline over the past decade. Similarly higher education departments in mathematics, science or engineering have seen difficulty in recruiting sufficient numbers of students to fill the increase in places as the higher education sector expands.

The TIMSS study by NFER and SCAA/OFSTED's *'Standards in Public Examinations'* study have considered changes in performance and the nature of mathematics education in England and Wales. The main findings of the TIMSS study for pupils aged 13 are that performance in mathematics is relatively weaker than science. Performance in 'data representation' and 'analysis & probability' was higher than the international mean, although performance in 'fractions & number sense', 'geometry', 'algebra', 'measurement' and 'proportionality' were below the international mean. Since the second international study the relative performance of mathematics has declined. For pupils aged 9, the TIMSS study suggests that standards in England and Wales are below the international average, particularly in 'number sense'.

The *'Standards in Public Examinations'* study found that at age 16 GCSEs had changed, leading to a broadening of mathematics content and skills, increasing use of formulae sheets, a **decline** in coverage of algebra, geometry and trigonometry and an **increase** in data handling, graphical representation of functions and matrix algebra. There has been a general increase in the structure of questions-this allows lower attainers better to demonstrate their understanding, but can reduce demand on most able candidates. In conclusion, the report stated that the standards shown by the majority of candidates at this level have probably risen, but the different emphasis on question structure and depth of treatment has led to standards falling for the most able. At age 18 the report found there had been changes in A level syllabus content but these had broadly been in balance. The most significant change had been to the style and structure of questions in most examination papers-in pure mathematics and mechanics questions had become generally shorter and mostly restricted to single topics, familiar contexts were used and algebraic manipulation was more basic than required for earlier questions. Questions were broken down into more steps, with prompts often given. Examinations were therefore less demanding, but an absence of question choice and increased time pressure has meant that candidates needed to be well versed in the whole syllabus. In conclusion the report stated that in pure mathematics standards at E grade had fallen. At A grade many aspects of performance were comparable, although candidates were not required to demonstrate as much competence in problem-solving, reasoning and algebraic manipulation.

Pre-16 Education & National Curriculum

Algebra

As noted above, there has been less emphasis on algebra in both pre-16 and pre-19 education. The RS/JMC Working Group report has considered in detail these changes and made recommendations on strengthening algebra education. The Society supports the broad recommendations of the Working Group, and stresses in particular the following points:

- There has been a change in emphasis in school algebra to focus more on activities which relate to discovering expressions and equations, such as pattern-spotting, and less on manipulating and simplifying algebraic expressions, and solving equations.
- The nature of problem solving in algebra seeks an emphasis on solving real world situations, relating algebra to pupils' informal methods and a de-emphasis of the role of symbols. The current trend is to introduce algebra within supposedly realistic contexts, especially around patterns of objects. Within these types of problems, the introduction of the algebraic modes of thinking and analysing tends to be delayed and is not viewed as central to the problem solving process.
- Precursory activities to algebra should be taught in both Primary and early Secondary schools. For example, such activities as working with relationships, expressing relationships involving more than one operation could be taught. Algebra should be introduced from the beginning of secondary school with more emphasis being placed on all aspects of algebra, as in countries such as France and Germany. Similarly, post-16 institutions could develop bridging courses in algebra for some students before they start A-level.

The Society recommends that these recommendations should be built in to the revision of the National Curriculum at the end of the moratorium on change recommended by Sir Ron Dearing. We are aware that the revised GCSE syllabuses, which were available from September 1996, have a greater proportion of algebra—we recommend that this increase in coverage is carefully monitored to ensure it is sufficient.

Other issues

Since 1994 there has been an increase in the number of students who have achieved Grade B from the Intermediate tier and have embarked on A level study. The Society recognises the difficulty of progressing to A level mathematics from Intermediate tier papers at GCSE, as the reduced syllabus will have very little coverage of key aspects such as algebra. Efforts must be made to encourage greater number of students to study for higher tier papers.

The criteria for school performance tables, with their emphasis on the proportion of students achieving grade A*-C, may well distort the choice of GCSEs studied. In mathematics this may be reflected in more students entered for intermediate tier papers to increase the chance of a grade C or B, with a consequent reduction in the number of students entering for the more demanding top tier papers. Alternative approaches to using the proportion of grade A*-C GCSEs could be considered. The Society hopes that further research will be undertaken on this issue.

With the development of the National Curriculum the time generally made available for mathematics has come under pressure. The recent emphasis on numeracy in Primary schools, and the use of a 'numeracy hour' recognizes this. The Society believes it is important that the time allocation for mathematics does not diminish further, particularly at Key Stage 4. We recommend that OFSTED and QCA monitor the amount of time allocated to mathematics at Key Stages 3 and 4.

Post-16 Education

The past two years have seen a major review of 16-19 education conducted by Sir Ron Dearing. Although the review looked at the general structure of education at this level, a number of recommendations were highlighted for mathematics. The Society is broadly supportive of the recommendations of the review, including the development of the Advanced Diploma for those students who want a broad-based study. However, the Society has made recommendations to change the criteria upon which the Diploma will be awarded as it is likely that as currently proposed it will not be appropriate for those students who opt for the study of both mathematics and science.

A major concern to the Society is the relative difficulty of mathematics and science at A level compared to other subjects. It is clear that this is widely perceived by students, and is one of the major deterring factors in recruiting students to mathematics and science A levels. Research by the A Level Information Service (ALIS) at Newcastle University and by SCAA has quantified the increased demand that mathematics and science place on students compared to other subjects; both found that mathematics and physical sciences are the most demanding subjects. The Society was pleased to see this problem highlighted in the 16-19 review, with the recommendation that marked discrepancies between subjects be addressed by levelling up the least demanding subjects. It is of great importance that this is addressed as soon as possible.

The Society welcomed the commitment to reviewing the common cores for A and AS levels in line with the general recommendations in the 16-19 review. The Society supported the move to increase the size of these cores to ensure a greater measure of consistency of material covered across different GCE boards. This will be particularly beneficial to higher education. We recommend that further consideration be given to increasing the focus on key mathematical concepts by reducing the breadth of material in A level mathematics syllabuses. It is of the highest priority that students have a fluent understanding of pure mathematics. If necessary, the amount of time spent on statistics and mechanics should be reduced to accommodate this. It is essential that in any revision of this kind that higher education, as major 'users' of students, should be fully involved in the process of syllabus change.

The 16-19 review recommended the reformulation of AS levels, including the suggestion that awarding bodies consider the case for a broader range and use of subjects at AS level. The Society warmly welcomed this, noting that such uses may include AS mathematics designed to serve the needs of other subjects, or encouraging more students to take mathematics without the commitment to a 'full' A level. However, the Society was most concerned at the timetable set for the revision of the A and AS common cores. Such fundamental change clearly needs the support of the whole education community, including schools and universities. Opportunities for considering such fundamental changes are unlikely to be repeated often, and it is important that this process is managed properly.

The 16-19 review also made recommendations on preparing students for A level mathematics, notably the development of an additional GCSE paper in mathematics. In the Society's comments on the 16-19 review this recommendation was rejected, not least because its implementation in schools would have been extremely difficult. The Society recommended that a clearer focus on fundamental mathematical tools at GCSE level would better prepare students for further study in mathematics.

It is important to recognize that higher education now takes in students with a wide ability range. It is right that it should, and it must make proper provision for the whole of this ability range. It is important, however, that all students feel intellectually challenged (in a way that, of course, is encouraging to students and not discouraging). It is thought that this is not the case at the moment for many students. Most serious, perhaps, is the concern that the most able students are not being challenged, and as a result are not stimulated towards achievements of which they are capable. It is important to challenge pupils at school, otherwise there will be a reluctance to be challenged in higher education.

Teacher Supply & Support

The Society has noted the difficulty with recruitment to initial teacher training in mathematics and science. It is clear that there is a lack of highly qualified teachers entering the profession. This point is a matter of real urgency and must be tackled quickly at national level. The Society continues to urge Government and its relevant agencies to address this matter. This problem manifests itself in various ways. Of particular concern is the use of non-specialist teaching mathematics in years 7-9; developing a good mathematical foundation at this age is clearly important.

Similarly, the support available to teachers through in-service training is declining. The role played by local authority advisors was considerably diminished in most authorities. The '20-day' INSET courses have been extremely valuable, yet DfEE funding for these has been withdrawn. Investing in the teaching profession is central to improving the quality of teaching and learning.

Regulatory and Awarding Framework

Many of the issues highlighted in this paper will require action to be taken at a national level. The Society is concerned that mechanisms to ensure wide consultation and the involvement of the education community, including higher education, are not in place. Our earlier concern over the rewriting of the A and AS cores is a good example where there was negligible debate on an issue of great importance. Too often short-term advisory groups have been established to advise the relevant agency on individual matters. These groups, by definition, can not represent all aspects of the education community. We are firmly of the view that more robust mechanisms should be put in place to allow for a fuller debate. Previously the Society has called for subject based advisory committees to be established within SCAA. The merger of SCAA and NCVQ presents an opportunity to reconsider these advisory mechanisms; the Society restates its recommendation for a permanent advisory committee for mathematics. The membership of such a group would be widely drawn ensuring full representation from all aspects of education. Furthermore, effective mechanisms must be established to ensure an overview of non-curriculum issues such as teacher recruitment and training, in-service support, etc.

Appendix A: References

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