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Teaching and learning geometry 11-19

Report of a Royal Society / Joint Mathematical Council working group

Summary

This report presents a summary of the findings of a working group established by the Royal Society and the Joint Mathematical Council to consider the teaching and learning of geometry in schools and colleges. The study was initiated following the publication of results of international educational comparisons, the 1999 revision of the National Curriculum for English schools 11-16, and at a time of several major policy initiatives in education.

The mathematical content for pupils following the National Curriculum in secondary schools in England is described under the headings of: *Number and algebra*; *Shape, space and measures;* and *Handling data*. However the term 'numeracy' has become increasingly used in place of mathematics in relation to school education. This is an unfortunate practice since it downplays two areas, algebra and geometry, which are of major importance in school mathematics. At school level, algebra can seem quite abstract and cerebral. On the other hand, there are clear links in geometry to the world of our senses and experience.

Geometry is of far reaching importance beyond the worlds of professional mathematicians and of mathematics teaching. Geometry is frequently used to model what we call the 'real world' and has many applications in solving practical problems. Geometry is making contributions to many important scientific developments such as the Human Genome Project, Buckminster-Fullerene research, and whole-body tomography. Through media such as film, television and computer games we encounter computer generated geometric images of great complexity, and children and adults alike derive pleasure from creating designs and patterns exhibiting geometric forms.

So geometry is an important subject, with wide applications and a long history. It deals with matters we find attractive and for which we have a strong visual capacity. On the surface, then, it would appear that geometry should be one of the easiest branches of mathematics to teach. But this is not the case - neither in England nor in much of the developed world. This Royal Society / Joint Mathematical Council study set out to identify why this is so.

A working group was established under the chairmanship of Professor Adrian Oldknow to undertake the study. Membership of the group comprised Dr Tony Barnard, Dr Richard Bridges, Professor Margaret Brown, Ms Sandy Cowling, Ms Caroline Dawes, Ms Margaret Dawes (Secretary), Professor Robin Forrest, Ms Jane Imrie, Dr Keith Jones, Professor Anthony Kelly, Ms Mary Ledwick, Dr Sue Pope, Dr John Rigby, Professor Chris Robson and Sir Christopher Zeeman. Observers to the group comprised Mr Richard Browne (Qualifications and Curriculum Authority), Mr Nigel Thomas (The Royal Society) and Mr Alan Wigley (National Numeracy Strategy).

The working group considered the rationale for a geometry curriculum, its possible content and issues concerned with its effective teaching. The report reflects the group's agreed views on the state of geometry teaching 11-19 and the major

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This summary can be found at www.royalsoc.ac.uk

issues needing to be addressed to bring about improvements. It is supported by additional materials, some of which are printed in the main report as appendices, and others of which are accessible from the Royal Society website at www.royalsoc.ac.uk These additional materials are intended to help illustrate some of the points made in the report, and to offer examples of approaches which might be taken by schools and colleges.

In order to help identify major issues raised, the report is structured around a number of agreed Key Principles. In the full report these are presented together with explanations, supporting arguments, and, where available, evidence. For this summary, however, we simply list them together with the working group's sixteen recommendations.

Overall, for mathematics 11-16, the working group concludes that the geometrical content of the new National Curriculum, with a few adjustments, forms an appropriate basis for a good geometry education. In order for this to be achieved however, considerable changes are needed in the way geometry is taught. It is vital that those working to improve mathematics education ensure that their work contributes significantly to improvements in geometry (as well as mathematics) teaching. Bringing about improvements in geometry teaching will require a significant commitment to a substantial programme of continuing professional development alongside the development of appropriate supporting materials.

For mathematics post-16 the working group concludes that there are insufficient opportunities for students to build on their 11-16 studies in geometry. Those concerned with curriculum design need to review the structure of post-16 qualifications in mathematics to ensure they provide improved opportunities for students to continue to study geometry. The provision of challenging and interesting geometry should help make mathematics a more attractive subject of study for more students. This is turn would contribute to overcoming the current shortage of those with good mathematical skills.

Key Principles

1. Geometry should form a significant component of the mathematics curriculum for all students from 11 to 19.

2. Any choice of curriculum should be underpinned by a rationale.

3. The geometry curriculum should maintain breadth, depth and balance, and be consistent with Key Principle 2 and the objectives in Recommendation 3.

4. Geometry should be given a higher status, together with a fair share of the teaching time available for mathematics.

5. Students in 16-19 education should have the opportunity to continue further their studies in geometry.

6. The assessment framework for the curriculum should be designed to ensure that the full range of students' geometrical knowledge, skills and understanding are given credit.

7. The most significant contribution to improvements in geometry teaching will be made by the development of good models of pedagogy, supported by carefully designed activities and resources, which are disseminated effectively and coherently to, and by, teachers.

8. It is a matter of national importance that as many students as possible fully develop their mathematical potential. Geometry, with its distinctive appeal, should make mathematics attractive to a wider range of students.

Recommendations

1. We recommend that curriculum and assessment specifications be reviewed to ensure that geometry forms a significant component of the mathematics curriculum for all students from 11 to 19.

2. We recommend that the title of the attainment target Ma3 of the National Curriculum be changed from 'Shape, space and measures' to 'Geometry'.

3. We recommend that the geometry curriculum be chosen and taught in such a way as to achieve the following objectives:

   a) to develop spatial awareness, geometrical intuition and the ability to visualise;
   b) to provide a breadth of geometrical experiences in 2-and 3-dimensions;
   c) to develop knowledge and understanding of, and the ability to use, geometrical properties and theorems;
   d) to encourage the development and use of conjecture, deductive reasoning and proof;
   e) to develop skills of applying geometry through problem solving and modelling in real world contexts;
   f) to develop useful Information & Communication Technology (ICT) skills in specifically geometrical contexts;
   g) to engender a positive attitude to mathematics; and
   h) to develop an awareness of the historical and cultural heritage of geometry in society, and of the contemporary applications of geometry.

4. We recommend that the current geometrical content of the English secondary school mathematics National Curriculum be regarded as a reasonable basis for an appropriate and rewarding geometry education for all pupils.

5. We recommend that the mathematics curriculum be developed to encourage students to work investigatively, demonstrate creativity and make discoveries in geometrical contexts so that students develop their powers of spatial thinking, visualisation and geometric reasoning.

6. We recommend that the mathematics curriculum be developed in ways which recognise the important position of theorems and proofs within mathematics and use the study of geometry to encourage the development of logical argument appropriate to the age and attainment of the student.

7. We recommend that the mathematics curriculum be developed to provide ample opportunities for students to use geometry for practical problem solving through mathematical modelling in both 2- and 3-dimensions.

8. We recommend that the geometry curriculum be developed to give greater emphasis to work in 3-dimensions and to make better use of Information and Communication Technology.

9. We recommend that the use of the word 'numeracy' in government publications and announcements be replaced by 'mathematics' to ensure that geometry is accorded its rightful position.

10. We recommend that geometry should occupy 25% - 30% of the teaching time, and hence a similar proportion of the assessment weighting, in the 11-16 mathematics National Curriculum.

11. We recommend that the total time allocated to mathematics 11-16 be monitored to ensure students spend at least 3 hours a week on mathematics, so that sufficient time is given to the teaching of geometry, and to other aspects of mathematics.

12. We recommend that a fundamental review be made of all 16-19 mathematics provision. This should include considering how:

    a) the structure and content of the current AS/A-level Mathematics and Further Mathematics specifications can better meet the needs of students and include a greater emphasis on geometry; and
    b) other post-16 mathematics qualifications, such as Free Standing Mathematics Units (FSMUs) and AS-level Use of Mathematics, can enable students to have the opportunity to continue their study of geometry.

13. We recommend that in the 16-19 curriculum the key skill 'Application of Number' be re-titled 'Application of Mathematics' and that the range of qualifying mathematical studies be broadened so that students continue their study of geometry.

14. We recommend that a review be made of the methods of assessment and examination used in mathematics at Key Stage 3, at GCSE and in post-16 qualifications to ensure that appropriate credit is given for the attainment of specific geometrical objectives.

15. We recommend that the relevant government agencies work together with bodies such as the mathematics professional associations represented on JMC, to provide a coherent framework for supporting the development of teaching and learning in geometry. This will involve:

    a) the recognition and development of good practice in geometry teaching through pilot studies and research;
    b) the design of programmes of continuing professional development and initial teacher education;
    c) the production of supporting materials; and
    d) the establishment of mechanisms to provide supporting resources, including ICT.

16. We recommend, in terms of mathematics in general, that:

    a) better publicity and information be provided to schools, students and parents about the career opportunities afforded by studying mathematics; and
    b) ways be sought to encourage schools and colleges to attract more students to study mathematics post-16, particularly at A-level.

A copy of the full report is available by sending an A4 self-addressed envelope (marked 'Geometry Report') to:

The Education Department, The Royal Society,
6 Carlton House Terrace, London SW1Y 5AG

Single copies of the report are available free of charge to schools, colleges and other educational institutions. For other organisations, or for multiple copies of the report, a charge of £15 per copy applies, and cheques made payable to 'The Royal Society' should accompany orders.