



# Mathematical Needs

The Mathematical Needs of Learners



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## EXECUTIVE SUMMARY

In 2009, the Advisory Committee on Mathematics Education (ACME, [acme-uk.org](http://acme-uk.org)) embarked on the Mathematical Needs Project to investigate how both the national needs and the individual needs of 5–19 learners in England can best be met by a curriculum, delivery policy and implementation framework. The project comprises two reports: one taking a ‘top-down’ approach by looking at the mathematical needs of Higher Education and employment; and a second taking a ‘bottom-up’ approach by examining the mathematical needs of learners. The overall aim of the project is to move to a situation where a full understanding of ‘mathematical needs’ is used to inform all the relevant policy decisions in England.

This report takes the ‘bottom-up’ approach and aims to identify what learners need in order to be successful and proficient in mathematics, to learn mathematics well, and to engage in mathematics lessons, and draws important conclusions and recommendations for a national policy.

ACME’s vision is for the mathematics education system to provide an environment within which all learners can be confident and successful in mathematics, with relevant policy decisions made according to their mathematical needs.

### Main findings

Mathematics is a highly interconnected subject that involves understanding and reasoning about concepts, and the relationships between them. It is learned not just in successive layers, but through revisiting and extending ideas. As such, the mathematical needs of learners are distinctive from their more general educational needs.

For mathematical proficiency, learners need to develop procedural, conceptual and utilitarian aspects of mathematics together. The full range of mathematical needs of learners is summarized in Table 1.

**Table 1 – The Mathematical Needs of Learners**

#### To be proficient in mathematics, learners need:

- procedural recall, accuracy and fluency in familiar routines.
- to develop procedural, conceptual and utilitarian aspects of mathematics together.
- the ability to interpret and use representations.
- a range of mathematical knowledge and experience.
- strategies for problem-solving and hypothesis-testing, including working with current digital technology.
- mathematical reasoning.
- appreciation of the purpose and usefulness of mathematics, and willingness to use it.

#### To learn mathematics well, learners need:

- to become aware of, familiar with, and fluent in connections in mathematics.
- to accumulate mathematical ideas.
- to have multiple experiences of mathematical ideas.
- time to develop the mathematical confidence to tackle unfamiliar tasks.
- to recognize the common ideas of mathematics.
- to learn how to listen to mathematical explanations.

#### To engage successfully in lessons, learners need:

- to read, talk and interpret mathematical text.
- to have a sense of achievement.
- to use feedback from tasks and results.
- to have good-quality explanations (images, representations, language, analogies, models, illustration).
- to have explanations that incorporate past knowledge, including familiar images, notations and mathematical ideas.
- teachers who understand the need to avoid unhelpful conceptions from particular examples, images and language.
- to base new learning on earlier understandings.
- teachers who push the boundaries of conceptual understanding.

#### Learners also need:

- teachers who have sound mathematical, pedagogical and subject-specific pedagogical knowledge.
- institutions and systems that take into account the needs of the different subjects in the criteria for qualifications, in methods of assessment and in accountability measures.
- school and college management who do not prioritize superficial learning for test results.

<sup>1</sup> With the raising of the school leaving age to 18, there will be many more learners in this age group.

Crucially, learners need to learn in an environment that recognizes the importance of these needs and is structured accordingly. This environment encompasses a wide range of factors, from the subject knowledge of those who are teaching them, the curriculum they are being taught and the mechanisms by which they are being assessed, to the school's own accountability structures, management and priorities.

International comparisons show that consistently important factors across successful countries include curriculum coherence and the quality of textbooks. This leads to two important points:

- the structure of the National Curriculum must reflect the nature of mathematics; it must present the sophisticated connections and relationships between key mathematical ideas in a non-linear fashion.
- teaching resources, such as textbooks, should focus on conceptual development rather than merely preparation for the next stage of assessment.

#### Several other points follow:

- young people need to be taught by a teacher who has a sound understanding of the connections in mathematics, and who can make good use of a curriculum that describes the links between concepts in this way.
- education policy – particularly changes to the National Curriculum – must be developed with mathematics specialists from all levels of education and at all stages.
- the assessment regime needs to incorporate all aspects of mathematical proficiency, not just those parts that are easy to test.

#### Many barriers exist to fulfilling ACME's vision:

- current school accountability systems encourage 'teaching to the test' and a procedural approach to mathematics.
- generic initiatives fail to take account of unique features of mathematics and impose 'one size fits all' policies.
- specialist teachers are in short supply, and opportunities to undertake funded study of mathematics through continuing professional development (CPD) are limited.
- there are mixed messages in society about the importance of mathematics.
- a lack of valued pathways post-16 that would allow all learners to continue studying mathematics up to the age of 18.

A simple message arises from this report: the mathematical needs of learners can be articulated, and must be taken into account alongside the needs of end users such as HE and employers in developing an education system that is truly fit for purpose. The barriers to meeting these needs can be overcome and must be tackled if England is to keep up with the progress being made by other countries.

## KEY RECOMMENDATIONS

### For policy-makers

- Subject-specific values, knowledge and methods of enquiry (including reasoning and application) should be upheld throughout the curriculum, assessment methods and materials, and teaching methods and resources. (Chapter 6 – Recommendation 7)
- The less easy-to-test aspects of mathematical proficiency should not be reduced to procedures in high-stakes assessments. The assessment regime should be revised to incorporate all aspects of mathematical knowledge and should encourage proficiency instead of short-term teaching to the test, which hinders understanding. (Chapter 3 – Recommendation 2)
- Resource production should be separated from the awarding organizations and resources such as textbooks should focus on conceptual development rather than what is necessary for the next level of assessment. (Chapter 8 – Recommendation 12)
- Pedagogy and assessment regimes should allow progressive development in mathematics and support positive attitudes to mathematics. (Chapter 4 – Recommendation 3)

### For the Department for Education when reviewing the National Curriculum

- Every review of the National Curriculum should take into account that doing mathematics involves a wide range of components and that learners need all of these components (Chapter 2 – Recommendation 1), including:
  - facts, methods, conventions and theorems.
  - mathematical concepts and structures.
  - connections between concepts.
  - notations, models and representations of situations within and outside mathematics.
  - symbols that are defined by formal rules of combination.
  - numerical, spatial, algebraic and logical reasoning within and outside mathematics.
  - mathematical ideas and contextual problems and applications.
  - deductions from axioms, hypotheses, generalizations and proofs.
  - generalizations from mathematical results and abstract higher-order concepts.
- Every review of the National Curriculum should take into account the fact that learning mathematics involves:
  - building on prior knowledge.
  - revisiting and extending familiar ideas.
  - frequent reappraisal and extension of understanding of key ideas.
  - developing a range of notations, facts and methods as tools for the future.
  - having a broad mathematical perspective.

Reviews should take into account the impact of past systems on pedagogy and learning. (Chapter 5 – Recommendation 4)

- ➡ Teaching should be done by knowledgeable teachers and presented as a conceptually coherent and cognitive progression of ideas that enables learners to develop all aspects of mathematical proficiency. This implies a curriculum review cycle that is long enough to develop a coherent, informed, package of assessment, textbooks and teacher knowledge. (Chapter 9 – Recommendation 13)
- ➡ The curriculum should (Chapter 9 – Recommendation 14):
  - be based on key mathematical ideas and how they are related in complex ways.
  - give opportunity for all learners to develop all aspects of mathematical proficiency (otherwise only what is tested will be taught) (Chapter 8 – Recommendation 11) and be based on conceptual development.
  - have sufficient detail and examples to avoid misinterpretation.
  - be reviewed at regular intervals, and be informed by societal needs, advances in mathematics and technology, and the current needs of learners.
  - ensure that mathematical thinking is developed, including problem-solving, reasoning, generalisation, proof and classification.
  - fully incorporate the mathematical capabilities, methods and questions that arise from use of all available technologies, especially those used in the workplace and those that are designed on mathematical principles. (Chapter 6 – Recommendation 8)
- ➡ When reviewing the National Curriculum, the Department for Education should note that:
  - a workable balance in the specification of the curriculum is essential. If the curriculum specifies too many irrelevant details, inexperienced and non-specialist teachers may not be able to decide what is important; if there is not enough detail, teachers may not be able to decide what to teach. (Chapter 9 – Recommendation 15)
  - two levels of statutory documentation would be helpful: (i) at policy level, describing the outline entitlement; (ii) at practitioner level, describing the essential ideas, components and proficiencies, and how they link together. (Chapter 9 – Recommendation 15)
  - the curriculum must show the sophisticated connections and relationships between key mathematical ideas in a non-linear fashion. (Chapter 9 – Recommendation 16)
  - it should also represent explicitly cross-curriculum ideas, such as measure and representation of data. (Chapter 9 – Recommendation 16)
- ➡ More work needs to be done to describe good pedagogy for mathematical continuity at the various transition stages to ensure learners' needs are met. (Chapter 7 – Recommendation 9)
- ➡ Materials that describe the use of mathematics outside and beyond school should be made available for those teaching and learning mathematics at all stages up to age 18. (Chapter 7 – Recommendation 10)
- ➡ School leaders should be informed about the subject-specific needs of learners and the implications for teaching. (Chapter 6 – Recommendation 5)

### **For teacher training and development**

- ➡ All teachers of mathematics should be entitled to subject-specific CPD. In particular, incentives and funding should be found for non-specialists to undertake subject courses at an appropriate level. Such courses should focus on key mathematical ideas, the latest research on teaching and learning, and the nature of mathematics. (Chapter 6 – Recommendation 6)



# 1. INTRODUCTION

**In 2009, the Advisory Committee on Mathematics Education (ACME, [acme-uk.org](http://acme-uk.org)) embarked on the Mathematical Needs Project to investigate how both the national needs, and the individual needs of 5–19 learners in England, can best be met by a curriculum, delivery policy and implementation framework. The project comprises two reports: one taking a ‘top-down’ approach by looking at the mathematical needs of Higher Education and employment; and a second taking a ‘bottom-up’ approach by examining the mathematical needs of learners. The overall aim of the project is to move to a situation where a full understanding of ‘mathematical needs’ of all the end-users of mathematics education – learners, universities and employers – is used to inform all the relevant policy decisions in England.**

This report takes the ‘bottom-up’ approach and aims to identify what learners need in order to be successful and proficient in mathematics, to learn mathematics well, and to engage in mathematics lessons.

ACME’s vision is for the mathematics education system to provide an environment within which all learners can be confident and successful in mathematics, with relevant policy decisions made according to their mathematical needs.

Each year a cohort of about 650,000 young people passes through England’s education system. Each of these young people will study mathematics from the age of 5 to the age of 16. They will need mathematics throughout their lives, in post-16 education, employment and everyday life, yet many are either turned off by the subject or do not feel confident using it. There is a general agreement that mathematics is crucial for economic development and for technical progress, but it is impossible to address the needs of employers and universities without taking into account the needs of young learners in schools and colleges. However, there is not always a clear understanding of what young people really need in order to progress in mathematics and to develop their mathematical potential. Without such an understanding, it is difficult to create policies in mathematics education that will enable young people to learn more mathematics and so increase the nation’s technological, economic and financial development.

## 1.1 Methodology

This project investigated what school-age students need to be effective learners engaged in mathematics, the experiences they need to continue to learn school mathematics, and the mathematical knowledge they need to be able to make informed choices.

### The approach used involved:

- collating evidence from high-quality research, and reports from reputable bodies, including those published by ACME which are themselves informed by research and practice.

- meetings with policy-makers, classroom teachers, and leading academics in mathematics education.
- collective thinking through seminars and workshops with the mathematics education community (see Appendix).

The report begins by explaining why the nature of mathematics requires us to identify specific mathematical needs of learners which are additional to their general educational needs (Chapter 2).

The outcomes of school mathematics education that are of value to learners, employers, universities, and other stakeholders, and which reflect the nature of mathematics are described in Chapter 3.

In Chapter 4 we draw on research relating to learners’ own perceptions of their needs and the reasons they give for a lack of engagement with mathematics. However, because learners cannot tell us everything about what would help them learn, we report, in Chapter 5, on how commonly agreed qualities of good mathematics teaching address learners’ needs, some of which are identified by the learners. In Chapter 6, we take this further by investigating how exceptional mathematics teachers teach. The aim here is not to identify ‘the right way to teach’ but rather to learn more about mathematical needs of learners by understanding what teachers are doing to fulfill them.

In Chapter 7 we examine the differences between learners’ typical experiences at different stages of schooling in order to draw attention to some of the additional social and emotional, as well as cognitive, factors that may affect their mathematical learning.

In Chapter 8 we reflect on the teaching and learning approaches, and the conceptual structure of the mathematics curricula in high-performing countries. This led, in Chapter 9, to a method for mapping the concepts that could be used to describe the connected nature of mathematics within a curriculum.

Throughout our investigation we came across barriers that impede progress towards ACME’s vision for mathematics education. These barriers, reported in Chapter 10, prevent teachers from doing more to address learners’ needs, and they prompt many of the recommendations of this report. We also point towards opportunities for overcoming those barriers.



## 2. THE NATURE OF MATHEMATICS

**A report such as this would be incomplete without a description of some of the features of mathematics that distinguish mathematical needs from general educational needs of learners.**

Mathematics requires understanding and reasoning about real and imagined objects, and is defined by a range of different kinds of knowledge, including<sup>1</sup>:

- facts, methods, conventions and theorems.
- mathematical concepts and structures.
- connections between concepts.
- notations, models and representations of situations within and outside mathematics.
- symbols that are defined by formal rules of combination.
- numerical, spatial, algebraic and logical reasoning within and outside mathematics.
- mathematical ideas and contextual problems and applications.
- deductions from axioms, hypotheses, generalizations and proofs.
- generalizations from mathematical results and abstract higher-order concepts.

These components link together in networks, hierarchies and layers. Until learners understand what a concept denotes, can use its notation, can reason about it, know some associated facts and theorems, and can represent it in equivalent ways, they cannot use it to solve complex problems or to help them learn further concepts. On the other hand, to understand the full meaning of a concept, they need to experience it in a range of situations within and outside mathematics.

For example, learners would have to know something about the meaning and characteristics of linear functions in order to know if a particular situation can be modelled using one, and this is more likely if they have seen how they relate to several situations, such as currency conversions or arithmetical progressions. Learners have to think about these links to be proficient in mathematics, so it is impossible to talk about desirable learning without also talking about the opportunities available to learn all the components listed. There are always a few students who will make these links for themselves, but others have to be helped to think in appropriate ways.

Crucially, procedural, conceptual and utilitarian aspects of mathematics develop together. There is much debate among researchers about whether it is better to teach methods before application, or to let methods arise from problem-solving (eg Leung and Li, 2011). Here we do not imply a particular order but insist that at all stages of their education learners need to bring together all aspects to do mathematics.

We use the word 'mathematics' throughout this document to include what is often called 'numeracy'. Numeracy can be defined as a quality of successful learners of mathematics (Coben, 2003) or as a proficiency which involves confidence and competence with numbers and measures, and the ability to solve number problems in a variety of contexts (National Framework for Teaching Mathematics, 1999).

However, whether it is seen as a quality or proficiency, numeracy involves all the components of mathematics described above and cannot be achieved merely by learning computations (Cockcroft, 1982). For this reason we cannot address learners' mathematical needs by imagining there is a definable subset called 'numeracy'.

### RECOMMENDATION 1

**1. Every review of the National Curriculum should take into account that doing mathematics involves a wide range of components, and that learners need all components including:**

- facts, methods, conventions and theorems.
- mathematical concepts and structures.
- connections between concepts.
- notations, models and representations of situations within and outside mathematics.
- symbols that are defined by formal rules of combination.
- numerical, spatial, algebraic and logical reasoning within and outside mathematics.
- mathematical ideas and contextual problems and applications.
- deductions from axioms, hypotheses, generalizations and proofs.
- generalizations from mathematical results and abstract higher-order concepts.

<sup>1</sup> This list is compiled from the range of questions posed in textbooks and examinations (not only current ones), tasks teachers set their students, reports of mathematical work from professional mathematicians, international curricula, and the experiences of workshop participants.



### 3. VALUED OUTCOMES FOR LEARNERS

**Students who are good at mathematics show qualities of persistence, independence, attainment and enjoyment (Watson, Prestage and De Geest, 2003). Learners need to be able to develop these qualities within the teaching they receive, but they could also be claimed for any subject.**

How these qualities translate into what is valuable in mathematics education was the focus of a seminar, organized by ACME, *Values and principles for effective learning of mathematics* (see Appendix). The conclusions of this seminar concur with research that was informed by lesson reports from 150 teachers and teacher trainers (NCETM, 2008) and international research into what defines mathematics proficiency (NCTM, 2000; Kilpatrick, 2001; Hiebert *et al*, 2003; Sturman *et al*, 2008; Askew *et al*, 2010).

The following characteristics were identified as being valuable outcomes of school mathematics:

- procedural recall, accuracy and fluency in familiar routines.
- opportunity to develop procedural, conceptual and utilitarian aspects of mathematics together.
- ability to interpret and use representations.
- a range of mathematical knowledge and experience.
- strategies for problem-solving and hypothesis-testing, including working with current digital technology.
- mathematical reasoning.
- appreciation of the purpose and usefulness of mathematics, and willingness to use it.

This view of mathematical proficiency matches the components required to do mathematics (see page 5) and the needs of employers and Higher Education as described in the 'top-down' approach to this project, *Mathematics in the workplace and in Higher Education*.

In order for these proficiencies to be taught, however, they need to be valued in curriculum documents, included in any high-stakes assessments (such as end of key stage tests), and allocated time and resources.

There is some evidence that such outcomes are valued in curriculum documents. For example, the 2007 National Curriculum for Mathematics (QCDA, 2007), emphasized process skills, including making choices about the mathematics and information to use, and being able to interpret, evaluate and communicate the outcomes of mathematical analysis. These were also described in the 1999 National Curriculum (QCA, 1999). The Free Standing Mathematics Qualifications (QCDA, 1999) and the current GCSE criteria value procedures, strategies, reasoning, use and application (QCDA, 2008 & 2009).

However, the current curriculum is seen as being fragmented, and GCSE and A-level examinations are dominated by routine procedures and familiar applications. There is strong agreement among teachers, educationalists and Ofsted inspectors that unless all aspects are assessed they will not be given significant teaching time and resource in schools and colleges.

While it is not yet clear to what extent GCSE assessment models can assess conceptual understanding as well as procedures and use, the current subject criteria state that about half the marks should be for technical fluency and the other half for candidates' ability to select and apply mathematics in context and solving problems. However, history contains several examples where mathematical thinking has been translated into procedures for ease of testing (eg GCSE mathematics coursework became a highly structured ritual), and teachers comment that there is a constant tension between curriculum aims and assessment.

Vigilance is needed to ensure that the less easy-to-test aspects of proficiency are not reduced to procedures in high-stakes assessments. Time and resources, and subject-specialist knowledge, need to be allocated to ensure the required outcomes are achieved and sustained. Moreover, Ofqual procedures need to be informed by subject specialists to ensure subject integrity.

In contrast to the situation in England, there is evidence that high-achieving countries in mathematics (PISA Frameworks 2003 & 2009) do value all the aspects relating to mathematics proficiency. Assessment instruments designed recently for the US, for example, encompass them on a continuum from novice, via apprentice, to expert standard (Gates Foundation, 2010).

Thus, including all items relating to mathematical proficiency in assessment models will not only ensure that they will be taught, but will also enhance England's international standing in mathematics.

#### RECOMMENDATION 2

- 2. Policy-makers should ensure that the less easy-to-test aspects of mathematical proficiency are not reduced to procedures in high-stakes assessments. The assessment regime should be revised to incorporate all aspects of mathematical proficiency and should encourage proficiency instead of short-term teaching to the test, which hinders understanding.**





## 4. THE LEARNER VOICE

**In talking about the mathematical needs of learners, we need to take into account their views. Within schools, 'learner voice' initiatives are an important mechanism for whole-school improvement. This is done by talking to learners about their experiences of teaching and learning, and changing practice as a result (eg Hargreaves, 2004; Rudduck and McIntyre, 2007).**

### 4.1 Primary school children

The recent Cambridge Primary Review (Alexander *et al*, 2010) collected views of children about their education. The authors report:

*Children were clear about what helped them to learn. They relished a challenge and being given opportunities for active hands-on kinds of learning. They wanted to feel able to succeed and to experience success. Praise from adults was important but so too was having a sense of personal satisfaction [...] children said they found it easier to learn when lessons were exciting or involved a variety of activities.*

The report found that young children recognized that their school day was divided up into different subjects with different purposes – in particular the '3 Rs' were seen as important and 'vital for future job prospects'. However, for these children mathematics was seen as 'necessary' rather than 'enjoyable', and even those who were successful in key stage assessments did not necessarily enjoy the subject.

An earlier study (Pollard and Triggs, 2000) found that in Key Stage 1 and early Key Stage 2, the core curriculum subjects, including mathematics, featured in children's lists of favourite subjects. However, by Years 5 and 6 these subjects had been replaced by subjects such as art and PE, where learners had more 'fun, activity and autonomy'. Subjects that were cited as the least favourite were those that were 'hard, difficult to succeed at, or offered the experience of failure', and included mathematics. However, the report also found that mathematics was a favourite subject both among higher-attaining students (who gained satisfaction in achieving success with challenging questions) and lower-attaining students (who gained satisfaction from answering closed questions with few writing demands). In comparison, middle-attaining students were those who least liked mathematics because they were fearful of failure and worried about the demands that the subject made on them.

### 4.2 Secondary school students

Recent studies on secondary school students' views about their experiences of teaching and learning in mathematics concur with the findings of the 1982 Cockcroft report, *Mathematics counts*, ie that most students do not regularly experience the approaches they say lead to effective learning.

For example, in a study of Year 8 students (Pedder and McIntyre, 2004), the researchers identified the following teaching and learning strategies that students said were the most effective:

- teachers need to engage actively with students' existing capabilities.
- contextualized learning would help students to connect concepts with things they were familiar with.
- tasks that foster a stronger sense of ownership would recognize their growing sense of independence and maturity.
- collaborative learning would promote greater discussion and working together on shared tasks.

However, an independent survey of secondary learners (QCDA, 2010) found that mathematics was cited as the subject that was least likely to:

- involve practical activities and group work.
- invite people from outside into school.
- make connections between different subjects.
- have teachers find out what prior knowledge students already had of a topic before teaching it.

A report by Ofsted (2008) based on its inspections of mathematics lessons and discussions with learners found that:

*Many pupils [...] described a lack of variety, which they found dull. Typically, their lessons concentrated on the acquisition of skills, solution of routine exercises and preparation for tests and examinations. Changes to this routine, such as investigations, practical activities and using new technologies, were seen as exceptions to this routine done at the end of term and were not thought of as 'real maths'.*

The Ofsted report also found that the secondary students they spoke to were 'ambivalent' towards mathematics; they knew it was important and wanted to do well in it, but they were rarely excited by the subject. These findings are supported by earlier research done with Year 9 students, whose experience of Key Stage 3 mathematics was reported to be 'TIRED' – tedious, isolated, rule and cue-following, elitist, and de-personalized (Nardi and Steward, 2003).

There is some evidence (Steward and Nardi, 2002; Brown *et al*, 2008) that secondary students link increased attainment in mathematics with enjoyment, though it would be a mistake to conclude that all high-attaining mathematics students enjoy the subject and want to take it beyond compulsory post-16 schooling (eg Boaler, Altendorf and Kent, 2011).

Thus predominant forms of teaching that learners encounter can lead to them coming across difficulties understanding mathematics (QCDA, 2010) and can lead to their disengagement from the subject (Nardi and Steward, 2003).

By the end of secondary school, learners' perceptions of mathematics are often of closed tasks and correct answers. They show increasing negativity towards mathematics, and associate their failures in the subject with low self-worth, even though they show an increasing understanding of the importance of the subject for their futures. They do not, in general, experience the breadth of the nature of mathematical activity, nor do they see themselves developing all aspects of mathematical proficiency.

In contrast, a study (Watson and De Geest, 2008) found that low-attaining students who were taught with a range of methods and high levels of interaction were not 'turned off' mathematics in the ways described here. Moreover, there is evidence that Functional Mathematics (QCDA, 2007-10) can lead to an increase in both motivation and engagement in mathematics (EMP stage 4 interim report, QCDA, 2007-10). There is also evidence that the pilot A-level Use of Mathematics has attracted new students into mathematical study because it gives students the opportunity to apply mathematics to the real world (Drake, 2011).

### 4.3 The effects of assessment regimes on learning

Declining attitudes towards mathematics from upper Key Stage 2 onwards are also linked to assessment strategies. In Year 6 mathematics is formally assessed through national tests, and throughout secondary schooling formal testing in mathematics regularly takes place.

At primary level preparation for, and pressure from, end of key stage tests reduces the popularity of the subject from its levels in previous years among all attainment groups, but particularly among middle-attaining children (Pollard and Triggs, 2000). A study of Year 6 pupils (Reay and William, 1999) revealed how the forthcoming assessments can leave pupils feeling of little or no worth:

*Pupil: I'm no good at spelling and [...] I'm hopeless at times tables so I'm frightened I'll do the SATs and I'll be a nothing.*

At Key Stage 4, when students prepare for GCSEs they are required to develop different strategies for coping in lessons which are different to those in previous years. An emphasis on 'pace' dictated by the teacher rather than having time to develop understanding forces learners to equate 'keeping up' with 'ability' (Harris *et al*, 1995).

In mathematics, learners are grouped by attainment – around 90 per cent or more of mathematics departments use this practice (William and Bartholomew, 2004). Different-ability sets within the same year group experience different teaching and learning approaches (Kutnick *et al*, 2006). There is some evidence (eg Arnot and Reay, 2004) that students' sense of self worth and engagement varies according to the set that they are in.

However, in general, students did not give any information about how they could best be helped to understand individual topics or even the general key ideas in mathematics. Their comments were dominated by how teaching methods, testing patterns and requirements affected their feelings, interest and self-worth in relation to mathematics.

In conclusion, these studies imply that it is not necessarily mathematics itself that is problematic, but rather the nature of the curriculum and the teaching methods and assessment regimes. Moreover, learners' views at both primary and secondary levels illustrate the dangers of a mechanistic approach to teaching and assessment.

### RECOMMENDATION 3

**3. Pedagogy and assessment regimes should allow progressive development in mathematics and support positive attitudes to mathematics.**



## 5. QUALITIES OF EFFECTIVE TEACHING

**Learners' understanding of the nature of mathematics is inevitably constrained by their experiences; they cannot describe their needs in terms of the wider mathematical perspective of understanding key concepts, being able to apply mathematical ideas, or being able to reason mathematically.**

**As one teacher said:**

**'There are some hard formalities in maths. Hard ideas that you have to get hold of. But they don't arrive naturally.'**

For these reasons, the 'learner voice' is a limited source of information about learners' mathematical needs.

Teaching and learning are closely linked. Teachers are the main instruments in helping learners with their mathematical needs. Thus the qualities of effective teaching can provide further insight into learners' mathematical needs. However, it is not ACME's intention to identify the 'right way to teach' but rather to learn more about the mathematical needs of learners by understanding what teachers do to fulfill them. It turns out that the forms of pedagogy that teachers use to develop complex mathematical proficiencies are closely matched with those that are known to be conducive to engagement, learning and effective teaching, and also with those that learners want.

### 5.1 Research-based guidance on effective teaching

Research-based guidance on effective mathematics teaching from the past three decades clearly points to the need for teaching that is knowledgeable, that draws on learners' understandings, involves discussions between teachers and students, engages all learners in a variety of complex tasks, and that presents mathematics as a subject with many components and proficiencies (Cockcroft, 1982; Smith, 2004; Williams, 2008; Day, Sammons and Kington, 2008; NCETM, 2008). For example, the 1982 Cockcroft report states:

*Approaches to the teaching of a particular piece of mathematics need to be related to the topic itself and to the abilities and experience of both teachers and pupils. Because of differences of personality and circumstance, methods which may be extremely successful with one teacher and one group of pupils will not necessarily be suitable for use by another teacher or with a different group of pupils. Nevertheless, we believe that there are certain elements which need to be present in successful mathematics teaching to pupils of all ages.*

*Mathematics teaching at all levels should include opportunities for:*

- exposition by the teacher.
- discussion between teacher and pupils, and between pupils themselves.
- appropriate practical work.
- consolidation and practice of fundamental skills and routines.
- problem solving, including the application of mathematics to everyday situations.
- investigation work.

The 1989 non-statutory guidance of the National Curriculum Council (NCC) offered two sets of principles that described the range of activities learners should experience. The first set suggests that activities in which young learners are most likely to engage with mathematics, and remain engaged should:

- where appropriate, use pupils' own interests or questions as starting points or as further lines of development.
- be delivered in a flexible order.
- be balanced between different modes of learning: doing, observing, talking and listening, discussing with other pupils, reflecting, drafting, reading and writing etc.
- where appropriate, involve both independent and cooperative work.
- enable pupils to develop their personal qualities.
- enable pupils to develop a positive attitude to mathematics.

The second set of principles focuses on the experiences learners need in order to be able to do the mathematics described in the first set of principles, and concur with the needs of employers and Higher Education, as described in the report *Mathematics in the workplace and in Higher Education*. The activities should:

- bring together different areas of mathematics.
- be balanced between tasks which develop knowledge, skills and understanding and those which develop the capability to tackle practical problems.
- be balanced between the applications of mathematics and ideas which are purely mathematical.
- be both of the kind which have an exact result or answer and those which have many possible outcomes.
- encourage pupils to use mental arithmetic and to become confident in the use of a range of mathematical tools.
- enable pupils to communicate their mathematics.

This description of effective mathematics teaching represents what students need to experience in schools and colleges in order to learn mathematics well and be able to achieve the valued outcomes. It matches aspirations for mathematics education throughout the world, even where overt aspects of pedagogy (such as how students are grouped, how lessons are structured, how much collaboration is used, how technology is used) are different.

Throughout this project there was agreement among teachers, researchers and other educators that these two sets of principles need to be restated regularly in documents about the teaching and learning of mathematics because they effectively define the learners' classroom experience and shape their view of the nature of mathematics as a subject.

While the proficiencies, desirable outcomes and types of activity in themselves do not define either good teaching or a curriculum, they do indicate that learners do not benefit from an over-simplified approach to teaching, which is currently the norm in many classrooms (Ofsted, 2008). Observational evidence from 192 schools, for example, found that classroom practice focuses on what is to be tested next and comes down to factual and procedural knowledge as well as predictable problem-solving techniques (Ofsted, 2008). 'Too much teaching concentrates on the acquisition of sets of disparate skills needed to pass examinations', the report states.

### 5.2 The effects of assessment regimes on teaching

In England, the national testing regime, coupled with a procedural emphasis in textbooks and national micro-management of planning and teaching, has contributed to some teachers adopting a mechanistic approach to teaching. This has been accompanied by raised test success and recent increases in A-level take-up of mathematics. However there is also evidence that this has led to a drop in enjoyment of the subject at Key Stages 2 and 3, despite an increase in the numbers of these learners appreciating its value as a qualification (Sturman *et al*, 2008).

Whether learners understand why mathematics has value is debatable since, for many, their experience of mathematics and what is required to pass examinations will not represent the full span of mathematical proficiency, nor will it address employers' needs (Ofsted, 2008).

Moreover, recent research has contested the idea that raised test scores indicate better mathematical understanding among today's students. In a study (Hodgen *et al*, 2010), researchers, using standardized assessment items, analyzed the results of 3000 14-year olds from 1976/7, and from 3000 14-year olds from 2008/9. The researchers found that over the 30-year period there has been:

- a fall in the proportion of 14-year olds who have strong understanding of algebra, ratio and decimals, and an increase in the proportion of learners who understand very little about these areas.
- within the middle band of achievement, there has been some increase in the proportion of students who understand decimals, no change with respect to understanding of ratios, and a decrease in the proportion of 14-year olds who are competent in algebra.

Thus, in terms of continued engagement and learning in mathematics, it is particularly worrying that fewer students today understand the key algebraic concepts of variable and generalized number, or have a well-founded understanding of ratio. The majority of items on decimals, algebra and ratio were found to be significantly more difficult for 2008 students. In contrast, students' understanding of whole numbers and the additive relation was found to have improved over this period.

These findings are supported by both the findings of the report *Mathematics in the workplace and in Higher Education* of this project, which makes clear the concern of employers and Higher Education tutors in these areas, and the Evaluating Mathematics Pathways (EMP) project carried out for the QCDA 2007-10, which highlighted the need to improve the algebraic skills of post-16-year olds.

There is therefore evidence of learners' needs being interpreted in a narrow way, the consequences of which are that learners have a narrow view and knowledge of mathematics.

### RECOMMENDATION 4

4. Every review of the National Curriculum should take into account the fact that learning mathematics involves: (i) building on prior knowledge; (ii) revisiting and extending familiar ideas; (iii) frequent reappraisal and extension of understanding of key ideas; (iv) developing a range of notations, facts and methods as tools for the future; and (v) having a broad mathematical perspective. Reviews should take into account the impact of past systems on pedagogy and learning.



## 6. USING EXCEPTIONAL TEACHING TO IDENTIFY LEARNERS' MATHEMATICAL NEEDS

Through networks of advisory teachers, ACME has identified teachers whose students consistently out-performed the statistical predictions in terms of test results. Advisory sources confirmed that this was not achieved by procedural 'teaching to the test'. Indeed, the majority of these teachers worked with students who would not have responded to a 'teach to the test' regime. These were learners whose engagement and motivation could not be taken for granted, yet the teachers were consistently able to help them achieve between one and two 'levels' higher than the expectations from the school improvement data used in their schools.

### 6.1 Learners' needs identified by exceptional teachers

In a seminar, *What do good teachers do?*, organized by ACME, a group of 'exceptional teachers' and representatives from mathematics education research community were asked:

*In the current national context, what can be learned from teachers who are achieving exceptional results, beyond statistical predictions, about the mathematical needs of learners?*

The question was not designed to elicit instructions about teaching but to provide an insight into the opportunities these teachers give their students in order to learn mathematics. Their practice reflects their professional perception of learners' needs.

The overarching aim of the seminar was to get beyond generic descriptions of good teaching – of enabling modes of classroom organization, or of formative assessment practices, or task type – to learn more about learners' needs in mathematics from those who work successfully day-to-day with students.

The key ideas that arose from discussions at this seminar, and from research reports about raising achievement, confirm that the principles outlined on page 9 of this report can be enacted within the current regimes and contribute to effective teaching. However, they do not tell the whole story of what learners need. All these teachers thought carefully and spent much time on ensuring that their students were involved in the lessons, made choices, felt comfortable about asking questions and making mistakes.

In addition, the teachers presented reports and examples of their practice at the seminar, and the project team identified common features that they use to fulfill learners' needs.

#### Learners need a sense of mathematical learning

1. Teachers provide frequent examples of the internal connections of mathematics to help learners make sense of the mathematics they use. To understand why they are being taught new ideas, learners

need to be able to fit different ideas together in a structure. The teachers were aware of, familiar with and fluent with connections within mathematics. This arises partly from personal knowledge and partly from their experience of teaching.

**Example A:** teacher recognizing the value of the grid layout for numerical multiplication, and building on it to multiply algebraic expressions and surds (and, later, complex numbers).

**Example B:** teacher pointing out that linear expressions contain variables of the first order by showing counter examples of quadratics and cubics 'to be met later'.

2. Teachers give learners the opportunity to accumulate a 'toolbox' of useful mathematical ideas. One teacher explained: 'All [my] students have their own "resource book". They don't have a textbook, they write all their notes in their own words in this book. This encourages them to take ownership of their learning.'

Teachers also demonstrate the use of past knowledge and ways of problem-solving themselves. This all takes time and requires giving learners multiple experiences to tackle unfamiliar tasks.

**Example A:** learners discussing when and why to use a number line to support their reasoning.

**Example B:** recognizing what kinds of situation might be modelled by exponential functions.

3. The teachers work explicitly on helping students learn how to listen to mathematical explanations. They need to know how to engage with symbols, examples, reasons, generalities etc. Teachers provide tasks that create a need to listen.

**Example A:** asking students to read aloud number sentences, or algebraic or graphical presentations and think about their meaning.

**Example B:** comparing the outcomes of two tasks that seem very different, but which unexpectedly (for the learners) generate the same equations and/or relations, and therefore need explanation.

#### Learners need teachers who support their conceptual development

4. Learners need to read, talk and interpret mathematical language to become familiar and confident with it when discussing mathematical ideas. The mathematics classroom is a place for extending vocabulary. As one teacher explained: 'I talk about the key words with the students and their definition and they work on being comfortable to use the words.'

**Example A:** expecting students to use the word 'divide' to describe quotient and partition, rather than everyday words such as 'sharing' or 'cutting up'.

**Example B:** explaining the focus of the lesson in terms of overarching big ideas, using mathematical language as well as everyday language.

5. Learners need to gain a sense of achievement in a mathematical environment. They gain this by being able to: check multi-stage tasks and operations for veracity and reasonableness themselves; see how their new skills can be applied in harder situations; and compare the results of their actions in different representations. They need rapid feedback (especially using self-checking methods) about what works and what doesn't in order to continue thinking, and this can be achieved in the way tasks are designed and set. Feedback from the teacher is also important, especially for development of self-esteem and input of new ideas.

**Example A:** testing division by multiplying.

**Example B:** predicting which vector will move an object to a given position and checking by using it.

6. Learners need good-quality explanations that include the use of powerful and extendable images, different representations, careful choice of language, isomorphic analogies, models and illustrations incorporating past knowledge, and familiar images, notations and mathematical ideas. One teacher explained: 'I include powerful images that can be worked on again and again; I try to pick out those that are most important.' Explanations can come from a variety of sources: teacher, textbook, new technologies, peers or through well-constructed tasks.

**Example A:** teachers choosing between images that give short-term gains but long-term confusion, such as apples and bananas for  $3a + 4b$ , and images that give longer-term gains, such as a  $3 \times a$  rectangle and a  $4 \times b$  rectangle.

**Example B:** when introducing function notation, relating it to the 'y=' notation, which might be more familiar, by showing when it does, and when it does not, fulfill the same purpose.

7. Learners are always generalizing their experiences. Consequently, they may build unhelpful conceptions from particular examples, images and language; they need to know that this is part of making sense of mathematics but can impede learning. They need teachers who anticipate common alternative conceptions that arise from their thinking, and help the students to overcome them. They also need teachers who try to understand less common alternative conceptions when they arise. If a task or new concept builds on prior experience, it is important to allow students to bring earlier understandings to the forefront of their minds for use and application. As one teacher said: 'You actually need to go back and forth', and another said: 'You can't plan ahead if you don't know what the students know.'

**Example A:** teacher 'cues' prior knowledge of right-angled triangles, ratio and angle in the two weeks before starting to teach trigonometry.

**Example B:** students know that  $n + 0 = n$ , so they often assume that  $n \times 0 = n$ . Teacher includes examples from time to time that lead to the explicit discussion of this assumption.

8. Learners need a variety of opportunities to think and talk about mathematical ideas so they can keep adjusting their understanding in the light of new experiences. Forming their own thoughts into words and hearing others talk helps learners organize and structure their ideas.

**Example A:** explaining subtraction when it involves physically taking objects away from a collection is different from explaining subtraction when it involves moving backwards on a numberline, and these need to be connected through speech.

**Example B:** thinking about and describing how the slope of a graph varies can provide a basis for representing acceleration.

9. Learners need to be asked good-quality questions that provoke mathematical thought and encourage deep, conceptual understanding.

**Example A:** 'Is it always, sometimes or never true that ...?'

**Example B:** 'Give me an example of a ... [some object] ... that has ... [an unusual feature]'.

### Learners need knowledgeable teachers and institutions

10. Points 1 to 9 confirm that the teachers' mathematical subject, pedagogical and subject-specific pedagogical knowledge are paramount. Teachers need to be confident and fluent in their mathematical ideas in order to engender similar confidence and fluency in learners.
11. There needs to be trust between learners and teachers, and between teachers and management. Managers need to understand what good mathematics learning comprises, and not apply generic judgments about what they see without understanding the mathematical context. Expectations of school and college management when monitoring teaching and progress often emphasize achieving measurable results through methods that lead to superficial learning, or direct funding towards particular groups of students, such as those on the borderline for school accountability measures (eg level 3/4 at Key Stage 2 or grade D/C at GCSE). Management support is needed for teachers to feel confident about adapting national guidelines to their context. As one teacher said: 'Teachers worry that it will lead to getting told off. It's a slow process to implement more varied teaching and you have to be willing to take the risk.'

## Summary of mathematical needs indicated by exceptional mathematics teaching

### To be proficient in mathematics, learners need:

- procedural recall, accuracy and fluency in familiar routines.
- to develop procedural, conceptual and utilitarian aspects of mathematics together.
- the ability to interpret and use representations.
- a range of mathematical knowledge and experience.
- strategies for problem-solving and hypothesis-testing, including working with current digital technology.
- mathematical reasoning.
- appreciation of the purpose and usefulness of mathematics, and willingness to use it.

### To learn mathematics well, learners need:

- to become aware of, familiar with, and fluent in connections in mathematics.
- to accumulate mathematical ideas.
- to have multiple experiences of mathematical ideas.
- time to develop the mathematical confidence to tackle unfamiliar tasks.
- to recognize the common ideas of mathematics.
- to learn how to listen to mathematical explanations.

### To engage successfully in lessons, learners need:

- to read, talk and interpret mathematical text.
- to have a sense of achievement.
- to use feedback from tasks and results.
- to have good-quality explanations (images, representations, language, analogies, models, illustration).
- to have explanations that incorporate past knowledge, including familiar images, notations and mathematical ideas.
- teachers who understand the need to avoid unhelpful conceptions from particular examples, images and language.
- to base new learning on earlier understandings.
- teachers who push the boundaries of conceptual understanding.

### Learners also need:

- teachers who have sound mathematical, pedagogical and subject-specific pedagogical knowledge.
- institutions and systems that take into account the needs of the different subjects in the criteria for qualifications, in methods of assessment and in accountability measures.
- school and college management who do not prioritize superficial learning for test results.

## 6.2 Implicit aspects of exceptional teaching

These exceptional teachers work within the current context and perceived constraints. Their own enthusiasm for mathematics enables them to maintain learners' interest in the subject.

One constraint is that the valued currency, from a learner's point of view, is a minimum of grade C at GCSE. As one teacher said: 'There is an expectation that a C grade is the grade to attain.' This can be obtained by giving only token attention to some of the key aspects of mathematics, such as using mathematics as a realistic problem-solving tool and developing mathematical argument, for example inductive and deductive reasoning and formal proof. Since neither of these are particularly necessary to achieve grade C, some teachers might choose not to include them in the mathematical experience of their students.

In contrast, exceptional teachers talked explicitly about how these aspects create purposeful mathematics lessons, and that problem-solving and mathematical reasoning are implicit aspects of their teaching. In particular, their students often learned mathematics through solving complex problems, as well as applying mathematics to problems in unfamiliar contexts. Without this variety and depth '...the treadmill that students go through can remove their confidence. They lose confidence in their maths abilities.'

Given that employers need learners to be able to apply and adapt their mathematical knowledge in unfamiliar contexts and reason mathematically (*Mathematics in the workplace and in Higher Education*), it would be a serious omission not to include these key ideas in the stated curriculum for all learners, however they are grouped and taught.

These exceptional teachers also commented on the lack of up-to-date technology in mathematics teaching. There is potential, by using mathematics-specific technology, to bring abstract mathematical ideas into the manipulable world (such as moving screen objects to substitute expressions as variables) and to experience the possible variation of mathematical objects through dynamic representations (such as conjecturing geometric relations and properties). While workplace uses of new technologies (such as structuring real data with spreadsheets, or creating and using databases and displays) might be learned when required in a particular context, the use of new technologies to advance mathematical knowledge is not embedded in classroom cultures; yet learners' outside lives and sources of knowledge are significantly influenced by current technology. One teacher said: 'The world of the student is IT. And then they go to a school where IT isn't part of the world. It switches them off.'

## 6.3 The need for subject-specific teacher knowledge

The qualities of good mathematics teaching are only sketchily described in generic terms in the national standards for qualified teacher status, but they are central to improving teaching and learning in mathematics. Learners need teachers who have a sound mathematical subject, pedagogical and subject-specific pedagogical knowledge. It is impossible to separate learners' needs from the quality of teaching and the knowledge and professional development of teachers.

For example, learning that images are important in explanations can be done quickly, but newly qualified teachers need to know the likely effects of using particular images: the hundred square or the number line for arithmetic; parts of circles, rectangles or lines for fractions; the

balance or the function machine for equations. Understanding these comparisons throughout the relevant curriculum takes time, and is unlikely to be achieved in school-based initial teacher education (ITE) courses or in generic CPD courses.

The need for subject-specific CPD is especially important for primary teachers who are the least likely to be specialists, yet have to lay the foundations for elementary concepts such as number, spatial reasoning and understanding relations (Nunes *et al*, 2009).

#### **RECOMMENDATIONS 5–8**

- 5. School leaders should be informed about the subject-specific needs of learners and the implications for teaching.**
- 6. All teachers of mathematics should be entitled to subject-specific CPD. In particular, incentives and funding should be found for non-specialists to undertake subject courses at an appropriate level. Such courses should focus on key mathematical ideas, the latest research on teaching and learning, and the nature of mathematics.**
- 7. Subject-specific values, knowledge and methods of enquiry (including reasoning and application) should be upheld throughout the curriculum, assessment methods and materials, and teaching methods and resources.**
- 8. The curriculum should fully incorporate the mathematical capabilities, methods and questions that arise from use of all available technologies, especially those used in the workplace and those that are designed on mathematical principles.**





## 7. TRANSITIONS

**Learners' mathematical needs can and will vary as they go through different phases of education. It is therefore important to look at learners' typical experiences at different stages of schooling to draw attention to some of the social and emotional, as well as cognitive, factors that may affect their learning of mathematics. In discussions between teachers of different phases throughout this project several factors emerged, which should be taken into account when considering the needs of learners.**

### 7.1 Social, emotional and cognitive factors

- Teachers' ideas about appropriate teaching styles and expectations of learners' behaviour often vary significantly in the 5–11, 11–16 and 16–19 age groups. Students moving from Year 6 to Year 7, for example, may find that their social experience in the classroom changes from one of responsibility to one in which they cannot do anything without permission, including getting up from their seat. Students moving from school to college may encounter lessons that are more like lectures.
- There is a lack of detailed communication between primary and secondary teachers, and between Key Stage 4 and Key Stage 5 teachers, about the learners' mathematical knowledge and understanding, and ability to apply these. National Curriculum test scores and GCSE grades do not provide this information. This can lead to unnecessary time spent re-teaching concepts that are not secure or glossing over concepts learners are presumed to understand.
- Students start new courses in Years 7 and 12 with a wide range of previous experiences. They will have been taught to apply mathematical techniques in different ways, and teachers and students may initially be using different 'dialects' of mathematical communication.
- Setting practices vary between schools and at different phases. Most (but not all) secondary schools teach in sets for Key Stages 3 and 4, while primary feeder schools may group pupils within classes. Alternatively, some primary schools might set pupils in Years 5 and 6, but the secondary school they feed into may teach in all-attainment groups in Year 7. A-level classes may have wide levels of prior attainment.
- Following examinations in May/June, pupils may not do mathematics regularly. The resulting lack of fluency may be interpreted as a lack of knowledge or understanding when they go on to their next institution.
- Students who take GCSE Mathematics early and who go on to study mathematics-related subjects post-16 may not get the opportunity to practice and develop their skills. Consequently, they may have gaps in their experience of key mathematical ideas. It is also not unusual for

a graduate primary teacher trainee to have studied no mathematics for at least six years when they start their training.

- Teachers may have little knowledge of how students learn mathematics at other stages of their development. It is difficult to teach mathematics at one level without a vision of where the mathematics is going at the next level and what has gone before.
- Students are used to ways of working with one teacher that may not be valued by another teacher.

Teachers need to be able to recognize what learners can already do and have done successfully in the past. Qualifications and records of coverage do not provide teachers with enough knowledge to ensure continuity for all learners. Teachers need to help learners interpret the mathematical language they are using in terms of individual earlier experience, and then build a secure, efficient understanding on that base.

Most of the transition issues identified are systemic or pedagogical rather than cognitive or developmental. In particular it is questionable whether a grade C at GCSE records readiness for all post-16 mathematics pathways, particularly in algebraic competence.<sup>2</sup> Choice of entry-tier and efficient question-spotting techniques can lead to students avoiding the topics that prepare them best for further study (EMP, stages 5 & 7 interim reports, QCDA, 2007-10).

### 7.2 School to university transition

Although ACME's remit does not extend to Higher Education, the transition from A-level mathematics to university-level mathematics can be particularly tricky for some learners and is therefore worth highlighting.

Research shows that some undergraduates with good qualifications have negative views of their learning experiences (Croft *et al.*, 2007; Solomon, 2007). These are often related to a limited view of mathematics. Students who experience pleasure and success in mathematics through positive reinforcement arising from consistently 'getting the right answer' are at risk of experiencing severe difficulty in the transition to university mathematics (Quinlan, 2009). And some learners whose success in mathematics had been based on getting definite answers turned off the subject during their first term in university (Daskalogianni and Simpson, 2002); not all were able to adjust to the new kinds of working and new views of mathematics. The shattering of their belief that there is an algorithm to solve any given problem is a shock (Ervinck, 1991). Importantly, the shift from performing techniques to proving properties is not one for which they are prepared, and they may feel overwhelmed and demotivated. This is also true for universities that attract learners with the best grades at A-level.

<sup>2</sup> ACME (2010) *Post-16 in 2016: Proposals for 16-19 Mathematics in anticipation of the review of qualifications scheduled for 2013 with resulting changes to be implemented from 2016*. London, ACME

For many learners at A-level, the core needs are to develop an intelligent interpretation of mathematical statements, basic techniques, confidence with mathematical notation, algebraic fluency, reasoning, proof, structure and classification. In addition, materials that elucidate the uncertainties, limitations and subjectivities of some beyond-school mathematics should be accessible for those teaching and learning mathematics. This would help teachers who may have little knowledge or confidence about the nature of beyond-school mathematics, or of current approaches in universities.

#### **RECOMMENDATIONS 9 AND 10**

- 9. More work needs to be done to describe good pedagogy for mathematical continuity at the various transition stages to ensure learners' needs are met.**
- 10. Materials that describe mathematics outside and beyond school should be made available for those teaching and learning mathematics at all stages up to age 18.**



## 8. INTERNATIONAL COMPARISONS

**It has been common practice nationally and internationally over the past 20 years to look at countries that are more successful in international comparative tests for ideas about better teaching. However, when cultural and social differences, out-of-school educational practices (such as tutoring), socio-economic differences within countries, attitudes to education and authority, and different curriculum coverage are taken into account, there is no evidence that any particular system of education is more successful than any other (Hiebert *et al*, 2003; Askew *et al*, 2010).**

A major factor that can influence the mathematical experience of learners is the curriculum. In a seminar, *Key Mathematical Ideas*, organized by ACME, a group of mathematics educationalists (see Appendix) analyzed the curricula of the following countries/regions to inform ACME's discussions about the structure of the curriculum: Finland, Singapore, New Zealand, Australia (Victoria), US (core standards), Canada (Alberta and Quebec), France, South Africa, Hungary, Hong Kong, China and Denmark. These countries/regions were selected because most are high-achieving in mathematics, several have some common characteristics with England (such as language and cultural diversity) and some have had recent curriculum reviews.

Mathematical content in terms of topics and the age at which they appear in the curricula was broadly similar across all countries/regions, but there were some differences in inclusion of statistical and probabilistic ideas, and higher pure mathematics such as complex numbers and matrices. The order and presentation of common content in documents and associated guidance and textbooks was variable (Askew *et al*, 2010). For example, algebraic manipulation was given more and earlier prominence in some, while algebraic expression was given early prominence in others. Other features (such as statements about curriculum aims, components of mathematical proficiency and guidance about good teaching) were similar, but the amount of detail varied considerably. There was no relation between the amount of detail and the international mathematical standing of a country.

International comparisons (Askew *et al*, 2010) however, show consistently that common factors across successful countries are:

- some mathematics topics are taught earlier than they are in England.
- they often start formal teaching later than England.
- they have a conceptually coherent and cognitive curriculum.
- textbooks are designed more for conceptual development than for coverage of examination content.
- a positive cultural value on mathematics for all.

Recent research syntheses published by the Nuffield Foundation (Nunes *et al*, 2009) identified desirable pathways of development of some key curriculum ideas (in England). While all versions of the National Curriculum for England and Wales illustrate knowledge of such progressions to some extent, the focus on 'levels' endemic in textbooks, school schemes of work and testing regimes obscures the need for learners to develop their understanding of key ideas over time, in a coherent pathway.

The development of a common ordered assessment system throughout school mathematics has led to a similarly ordered and fragmented approach to the curriculum in which mathematics is described as a sequence of horizontal levels, rather than a collection of pathways in which key ideas develop over time.

### RECOMMENDATIONS 11 AND 12

- 11. The curriculum should give opportunity for all learners to develop all aspects of mathematical proficiency and be based on conceptual development.**
- 12. Resource production should be separated from the awarding organizations, and resources such as textbooks should focus on conceptual development rather than what is necessary for the next level of assessment.**



## 9. MAPPING A CURRICULUM FOR LEARNERS' MATHEMATICAL NEEDS

**ACME believes that key ideas in the curriculum should be presented in terms of learners' needs in order to support their use and enjoyment of mathematics and their progression to the next stage of mathematics – an enabling prospective purpose – rather than being represented in terms of the next test – a limiting retrospective purpose. Moreover, ACME believes that there is a need for significant work to be done to describe the development of mathematical ideas throughout formal education from the learners' point of view, and for teachers whose only experience has been the current 'level' approach to teaching.**

The National Strategy published progression maps ([http://nationalstrategies.standards.dcsf.gov.uk/secondary/intervention/progression maps/mathematics](http://nationalstrategies.standards.dcsf.gov.uk/secondary/intervention/progression%20maps/mathematics)), which have provided some insight into ways of doing this, but the maps were limited because they were presented in a linear format based on levels determined by the current curriculum and its assessment.

A linear curriculum that lists topics and skills, which are tested in order, without elaboration about the complex ways in which concepts are learned, can lead to a fragmented, incoherent learning experience in which students are taught new ideas before they have developed sufficiently deep experience of earlier ideas. For example, naive notions of measure as 'that which can be measured with a ruler' have to be transformed into a general awareness of how quantities that are not one-dimensional are measured. Without this awareness, learners may not understand ratios of length, area and volume and their applications in science (Nunes, Bryant and Watson, 2009). Conversely, careful teaching of ratio can itself enhance and embed this sense of dimensionality, and it can then be developed in science and other contexts – but at some stage, in some way, the link has to be made, or progress in understanding will stall.

Topics that become more important as students' lives and employment prospects change need to be included in the curriculum, which means that other topics need to be removed. This is an argument for continual curriculum review to take place which is informed by societal needs and advances in mathematics and technology, and which allows students to make informed choices. ACME is particularly concerned about the sketchy inclusion of modelling and the mathematics associated with risk in many international curricula, and also in England's curriculum. A further aspect of mathematics that is crucial for design is the ability to create, test and use algorithms. These are not aspects of mathematics that will emerge from international curriculum comparisons or international tests, yet they arise strongly in our report on *Mathematics in the workplace and in Higher Education*.

Teachers, especially non-specialist mathematics teachers, are inevitably tempted to teach exactly what is required for the next test or next level instead of laying the foundations for understanding overarching

mathematical ideas. Teachers often claim they do not have time to teach breadth and depth because of pressure for coverage and test results. Lack of specialist knowledge can also make it harder for teachers to understand the connections and relationships between key mathematical ideas. This makes mathematics harder to learn for many learners, because they do not have the opportunity or the required knowledge to make sense of fragments of mathematical information within the broader context of mathematics and its applications.

### 9.1 The structure of the curriculum

In a seminar, *Key Mathematics Ideas*, ACME brought together a group of professionals who were experienced in mathematics, children's learning, curriculum design, mathematics teaching and assessment to address the following question:

*What are the possible ways in which the content can be arranged in order to promote an understanding of the connections and progressions within mathematics?*

The outcomes of this seminar are presented as recommendations 13–16 of this report. It is worth noting that the Nuffield-funded research synthesis *Key Understandings in Mathematics* (Nunes, Bryant and Watson, 2009) and the collective wisdom and experience of the working group offer broadly similar knowledge of appropriate links and progression.

#### RECOMMENDATIONS 13–14

- 13. The curriculum should be taught by knowledgeable teachers and presented as a conceptually coherent and cognitive progression of ideas that enables learners to develop all aspects of mathematical proficiency. This implies a review cycle that is long enough to develop a coherent, informed package of assessment, textbooks and teacher knowledge.**
- 14. The curriculum should:**
  - be based on key mathematical ideas and how they are related in complex ways.
  - include all relevant components of mathematics and all aspects of mathematical proficiency (otherwise only what is tested will be taught).
  - have sufficient detail and examples to avoid misinterpretation.
  - be reviewed at regular intervals, and be informed by societal needs, advances in mathematics and technology, and the current needs of adolescent learners.
  - ensure that mathematical thinking is developed, including problem-solving, reasoning, generalization, proof and classification.

## RECOMMENDATIONS 15–16

### 15. Amount of detail

- A workable balance in the specifications of the curriculum is essential. If the curriculum specifies too many irrelevant details, inexperienced and non-specialist teachers may not be able to decide what is important; if there is not enough detail, teachers may not be able to decide what to teach.
- Two levels of statutory documentation would be helpful: (i) at policy level, describing the outline entitlement; (ii) at practitioner level, describing the essential ideas, components and proficiencies, and how they link together.

### 16. Relationships, links and layers

- The curriculum must show the sophisticated connections and relationships between key mathematical ideas in a non-linear fashion.
- It should also represent explicitly cross-curriculum ideas, such as measure and representation of data.

To construct examples of this approach, as proof of concept, participants then addressed the question:

*What are the facts, laws, relationships, distinctions, representations and deep connections that learners need to understand in order to progress in mathematics?*

The aim here was not to produce a finished model, but to provide exemplars to show that a non-linear presentation of complex ideas is possible and could be used by teachers to inform their schemes of work. More work needs to be done to ensure that the pathways are coherent, informed and fit together for a whole curriculum.

Two examples were worked through: (i) the development of multiplicative reasoning; and (ii) understanding measurement. Figure 1 shows the basic model. The aim is to present the curriculum in terms of relationships between key mathematical concepts and ideas that would clearly identify and respond to learners' mathematical needs. The curriculum would be organized around the 'big mathematical ideas', and would relate to the learners' progression and the ideas that they need to understand and progress further in mathematics. In an electronic format, it is relatively straightforward for the curriculum to be presented in a number of 'layers' that address the variety of depths and knowledge with which teachers and other users approach it.

### 9.2 Multiplicative reasoning

The first example presents a way to look at learners' mathematical needs in relation to multiplicative reasoning. Figure 2 identifies key concepts, which become mathematical tools, and shows connections between them. The right-hand margin lists contexts and mathematical practices associated with multiplicative reasoning. The top margin lists the superordinate mathematical ideas (such as functions) in which multiplicative reasoning is embedded.

Learners need to understand many meanings and models to make progress and use their understanding in later concepts that depend on multiplicative reasoning. To develop the full range of proficiencies, they need to use multiplicative reasoning in the ways listed in the right-hand margin. To develop further mathematical understanding, they need to see that multiplicative reasoning is embedded in the ideas listed in the top margin.

The shift from additive to multiplicative reasoning is a key idea that takes time and multiple experiences. The progression illustrated in Figure 2 is that sharing, comparison and correspondence provide a basis

Figure 1 A model for mapping a curriculum for learners' mathematical needs

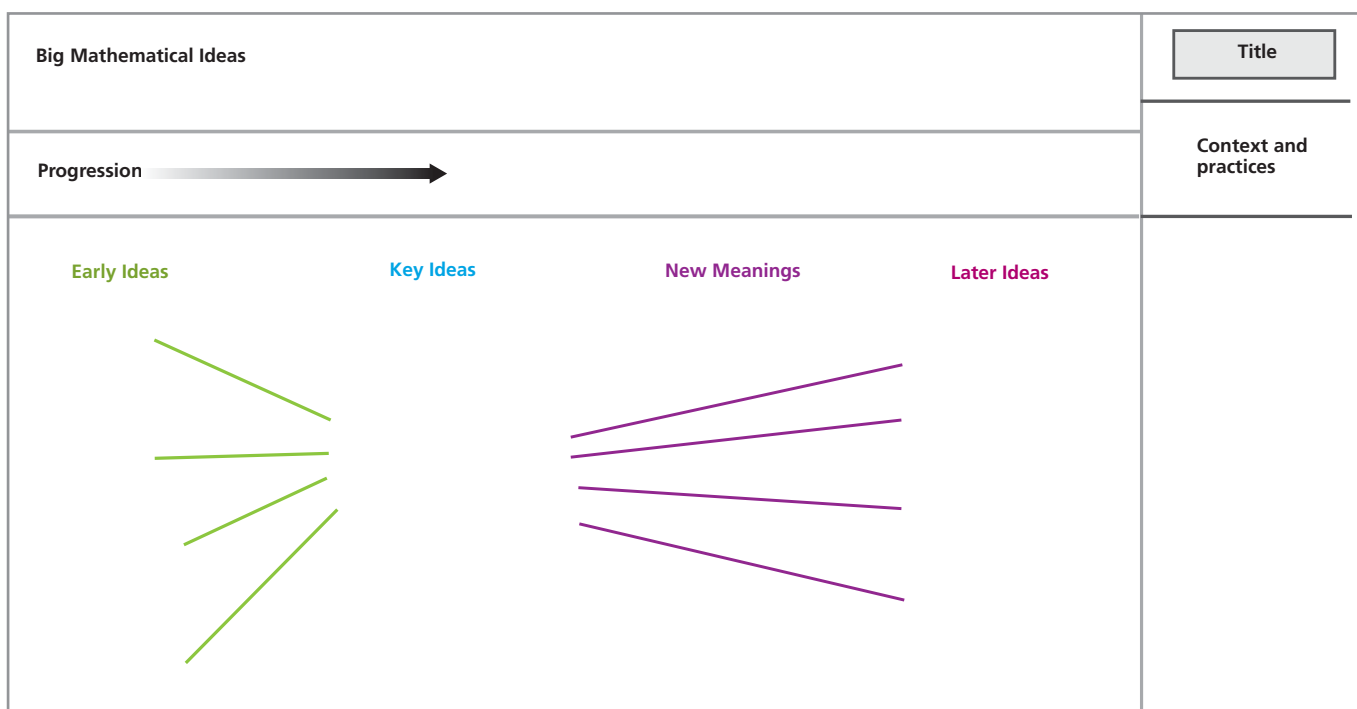
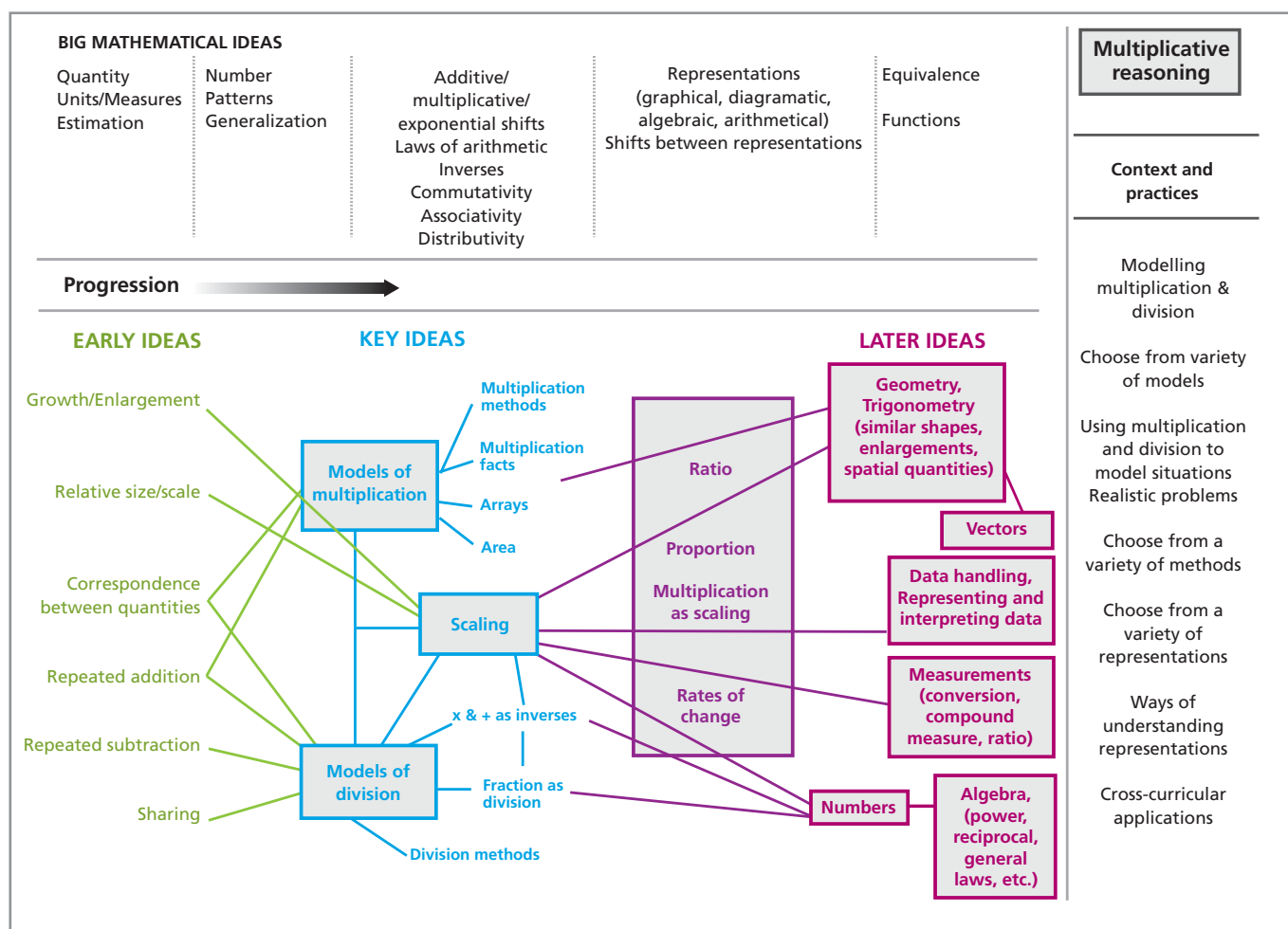


Figure 2 A model for mapping multiplicative learning



for multiplicative reasoning, as well as additive reasoning, which provides a limited basis. These elementary understandings generate different models of multiplication and division, and learners need to develop new understandings about relations such as scaling, inverse relations and fractions as division. The purple lines show how these new understandings relate to later concepts, for which the elementary ideas of repeated addition and sharing are totally inadequate.

**Key conceptual idea:** all numbers can be connected, related and compared through multiplicative relations.

### 9.3 Measurement

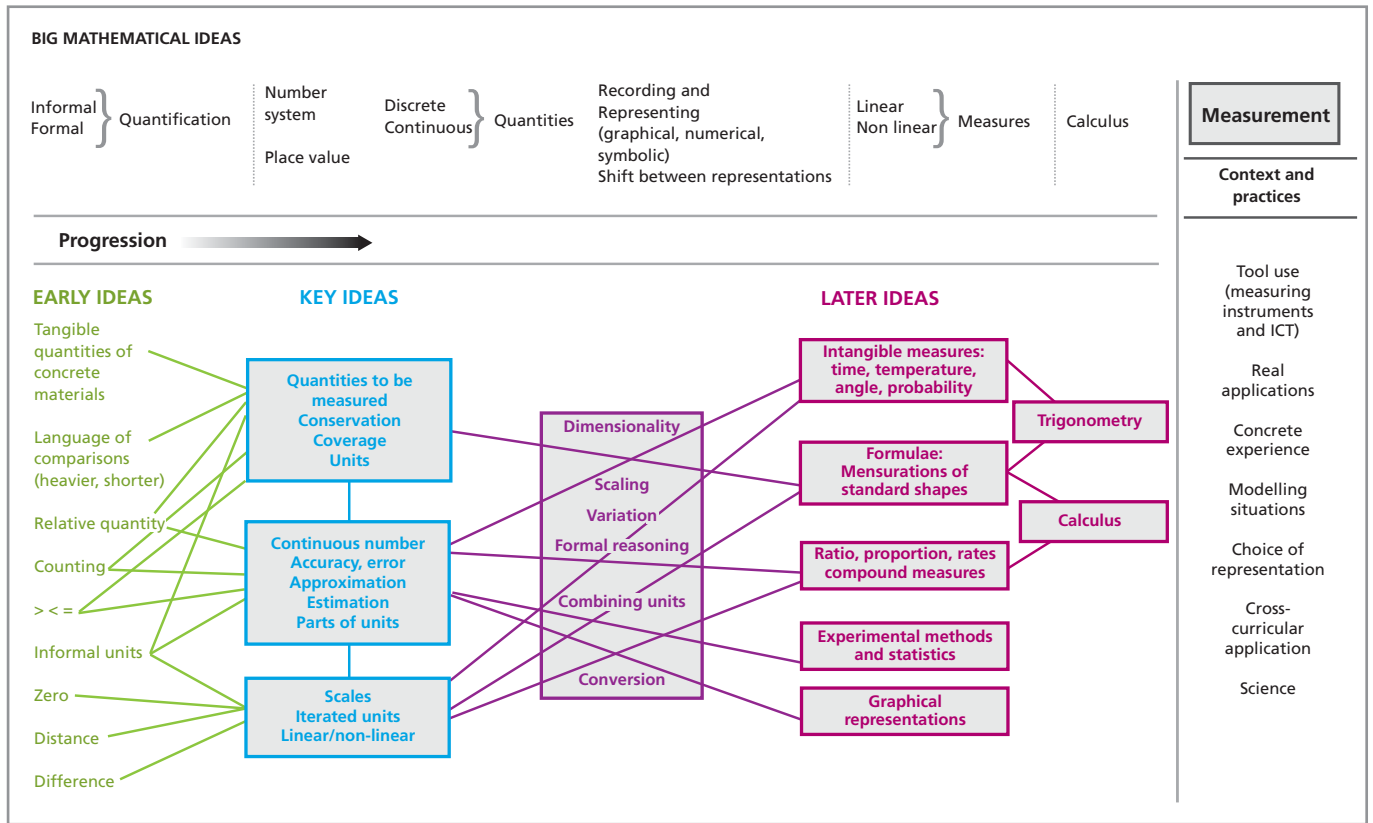
This example presents a way to look at learners' mathematical needs in relation to measurement. Figure 3 identifies the conceptual basis for measurement and how understanding measure impacts on other mathematical ideas. The right-hand margin lists general mathematical contexts and mathematical practices related to measure. The top margin lists the superordinate mathematical ideas, such as ratio, in which measurement is embedded.

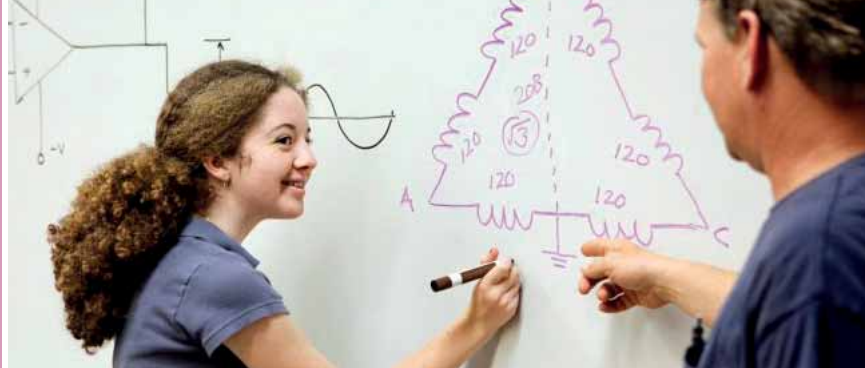
Measurement can be seen as a method for describing quantities. It requires: appreciation of conservation; understanding the nature and dimensionality of the quantities to be measured; understanding continuous number; understanding units and their iterative use; and knowing how to use the relevant tools. Learners have to understand how measurement relates to counting and scaling. The shift from discrete to continuous number is a key idea that takes time and multiple experiences. The green lines show these connections. The purple lines show connections to other mathematical ideas. Ratio is embedded in measurement since measuring compares units to other quantities. Compound measures, such as kilometers per hour, are rates that relate to gradients of functions.

Applications are manifold. As with multiplicative reasoning, to develop the full range of proficiencies, learners need to use measurement in the ways listed in the right-hand margin. To develop further mathematical understanding they need to see that measurement is embedded in the ideas listed in the top margin.

**Key conceptual idea:** measurement involves ratio, understanding of linear and non-linear quantities, continuous number, iterated units, compound units and tool use.

Figure 3 A model for mapping measurement





## 10. BARRIERS AND OPPORTUNITIES

Throughout our investigations we identified barriers that could impede progress towards ACME's vision. These were described repeatedly throughout the two years of the project by many people, and especially by teachers and heads of departments. These barriers prevent teachers from doing more to address learners' needs. The exceptional teachers said that they were fortunate in having the support of management in their schools to work in ways that were not always focused on short-term test results.

All of the identified barriers are addressed by the recommendations in this report. We have also identified opportunities that currently exist that could also help to overcome these barriers.

### POLICY, CURRICULUM AND THE STRUCTURE OF THE SYSTEM

**Barrier.** Current school accountability policy encourages 'teaching to the test', early entry, a focus on particular groups, and a procedural approach to mathematics. This can be exacerbated by school leaders misapplying generic quality control procedures in order to address perceived outside demands.

#### Opportunities:

- investigate international practices in accountability.
- tests should probe deeper understanding and application (eg the new GCSEs, especially the double mathematics option).
- prioritize diagnostic assessment to inform teaching (eg catch-up for younger children; and assessment materials which have been produced in Nottingham University for use in the US).
- clarify Ofsted subject inspection criteria to include a focus on expanding mathematics beyond procedures.
- utilize the NCETM<sup>3</sup> elaboration of TDA<sup>4</sup> standards for mathematics teaching (see [www.ncetm.org.uk/tda/#](http://www.ncetm.org.uk/tda/#)).
- the National College for Leadership of Schools and Children's services could emphasize differences in subject pedagogy.

**Barrier.** Generic policy initiatives avoid the fact that there are inherently difficult concepts in mathematics, which require knowledgeable intervention, time and multiple experiences. Teachers are often not experienced in curriculum development.

#### Opportunities:

- subject community-driven curriculum development.
- rethinking of qualifications and assessment structures.

- encouragement of local professional networks to enhance expertise, responsive to links with employers and Higher Education.
- NCETM support.

**Barrier.** Use of mathematics-enhancing technology is not universally embedded in school mathematics, and teachers have limited time and facilities to develop its use.

#### Opportunities:

- NCETM report published September 2010.
- JMC report to be published 2011.
- develop smart use of new technology for teaching and learning mathematics through local and virtual networks.

### RESOURCES

**Barrier.** Some text books and other teaching resources offer very limited opportunities for conceptual development.

#### Opportunities:

- consider whether national standards for textbook and resource design might enhance quality.
- develop teacher knowledge through ITE and CPD to move beyond published resources.

### TEACHING / LEARNING / CPD

**Barrier.** Some teacher subject knowledge is limited; the necessity and opportunity to undertake funded study of mathematics knowledge and pedagogy are limited in ITE and CPD.

#### Opportunities:

- NCETM provides information and accreditation for subject specific CPD opportunities.
- Mathematics Specialist Teachers (MaST) courses.
- Chartered Mathematics Teacher accreditation (CMaST<sup>5</sup>).
- courses for non-specialist teachers at Key Stage 3; expansion of subject association activity.
- evidence on how subject specialism impacts on teaching and learning soon to be published by SCORE<sup>6</sup> in summer 2011.

**Barrier.** Discontinuities of curriculum coverage, experiences, expectations and assumptions over the range of transition points between Key Stages 2 and 3, Key Stages 4 and 5, and beyond. These can be exacerbated by the influence of high-stakes assessment.

<sup>3</sup> National Centre for Excellence in Teaching Mathematics

<sup>4</sup> Training and Development Agency for Schools

<sup>5</sup> The CMaST designation is incorporated within the Royal Charter of the Institute of Mathematics and its Applications and is awarded by the Chartered Mathematics Teacher Registration Authority. See [http://www.ima.org.uk/membership/becoming\\_chartered/chartered\\_mathematics\\_teacher.cfm](http://www.ima.org.uk/membership/becoming_chartered/chartered_mathematics_teacher.cfm), accessed 26 May 2011.

<sup>6</sup> Science Community Representing Education



**Opportunities:**

- develop common understandings of mathematical priorities and good subject-specific pedagogy (eg teachers work together in clusters; mutual observations across all transitions).

**Barrier.** Lack of knowledge, at all levels, of the key mathematical ideas and how they develop.

**Opportunities:**

- teacher development as above; subject-specific expertise in Ofqual, Ofsted etc.; professional and research journals.

## PARTICIPATION AND PERCEPTIONS

**Barrier.** Tensions between increasing demand for post-16 experience in mathematics, and policies and practices that limit the study of mathematics post-16.

**Opportunities:**

- ACME Post-16 in 2016 report.
- post-16 teacher development via local networks.

**Barrier.** Mixed messages in society about the importance of mathematics for an educated person.

**Opportunities:**

- work by the subject community and employers to enhance awareness of the excitement and power of mathematics, especially in a technological society.
- increase in books, films and TV programmes featuring mathematics.

## CONCLUSION

Throughout this project we have identified various mathematical needs of young people that must be fulfilled in order for them to progress, enjoy and acquire mathematical confidence. The fulfillment of these needs would be a step further towards achieving ACME's vision for mathematics education.

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