THE ROYAL SOCIETY

Mathematics for the T Level Qualifications:
a rationale for General Mathematical Competences (GMCs)

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Summary

The Royal Society's Advisory Committee on Mathematics Education's (ACME) Post-16 Contact Group proposes that the mathematics in T Levels and its assessment is developed in line with a framework of General Mathematical Competences (GMCs). These have been designed with the purpose of enabling students to engage with mathematics in ways that capture the essence of working mathematically in vocational contexts.

The GMCs acknowledge difficulties often experienced in the transfer of knowledge from one domain to another and are specifically informed by, and sensitive to, research into the use of mathematics in workplaces. Furthermore, they provide a common structure around which mathematics for *all* T Levels can be specified whilst allowing for adaptation that ensures authenticity of student experience in *each* T Level.

Consequently, each GMC:

- applies in many situations and is sufficiently general to be useful;
- organises a substantial body of mathematical knowledge (concepts, skills and understanding);
- emphasises the use of mathematical models that provide insight into real (and complex) situations and problems that are simplified by making a number of assumptions.
- ensures experience of using appropriate technologies in ways that reflect how mathematics is used in workplace contexts and situations.

Each general mathematical competence stands independent of the 'level' of the mathematics being used. Hence, level 2 and 3 technicians as well as graduate technicians might, for example, all be 'processing data' but the complexity of their work could vary considerably as could the difficulty of mathematical working required.

A General Mathematical Competence is a 'boundary object' between the world of work and education in that it clearly has meaning in each community, albeit with different emphases. Each GMC provides opportunity for engaging students in both authentic vocational activity and meaningful and purposeful learning and application of mathematics.

This document sets out a framework of ten GMCs:

- Measuring with precision
- Estimating, calculating and error spotting
- Working with proportion
- Using rules and formulae
- Processing data
- Understanding data and risk
- Interpreting and representing with mathematical diagrams
- Communicating using mathematics
- Costing a project
- Optimising work processes

Curriculum implementation and assessment based on GMCs should be supported by an iterative programme of design research that secures high levels of validity, credibility and authenticity. It is likely that a hybrid model of assessment that includes both timed-written and portfolio assessment will best achieve this in ways that ensure alignment of curriculum, assessment and learner experience.

Mathematics for the T Level Qualifications: a rationale for General Mathematical Competences (GMCs)

1 Introduction

This document sets out a way of addressing an important issue raised by Professor Sir Adrian Smith in his 2017 review of post-16 mathematics[1]. In relation to the mathematics in technical education routes Smith said

> “Defining the appropriate mathematics for each of the technical routes is likely to be complex. The mathematics should be designed to reflect the requirements of the relevant occupations, wider society and the emerging economy. It needs to be coherently structured, taught and assessed.” (para. 134)

He proceeded to recommend that “The Institute for Apprenticeships should work with the Royal Society Advisory Committee on Mathematics Education (ACME) to ensure appropriate expert advice is available to the panels of professionals developing technical routes.”

Following preliminary work undertaken by Professor Geoff Wake (University of Nottingham) on behalf of the Gatsby Foundation, ACME’s Post-16 Pathways Contact Group has developed a particular design for mathematics in the T Level curriculum specifications that addresses the points above. The following sets out a rationale for the proposed framework of ten General Mathematical Competences that ‘coherently structure’ the mathematics in the T Levels.

This document provides background and contextual information appropriate to understanding mathematics education in and for work. It offers an overview of research on the use of mathematics in workplace settings; summarises a number of frameworks that structure mathematics that prioritise application; provides a theory-informed rationale for the proposed curriculum design, exemplifies how the GMCs might interact with T Level design, and advises on how best to assess students’ general mathematical competences.

2 Background

The teaching and learning of mathematics in pre-vocational and vocational programmes, including apprenticeships, provides a major challenge to teachers and learners. Fundamental to the problem is how to best ensure the development of mathematical competence for the ‘near and now’ demands of students/apprentices but also for their future as competent workers equipped with competencies that will be appropriate and relevant for unknown future workplace practices. Any curriculum has to ensure general preparation for a wide range of specific trajectories through quickly changing workplaces.

In this setting there have been different approaches over time. At one end of this range mathematics may be *embedded* in different vocational qualifications to the extent that it almost disappears. A middle position may take a form of *integration* where the mathematics is more or less explicit and at the other end of the range there may be separate units that can be certificated. This latter approach tends to be in the most technically and mathematically demanding vocational areas such as engineering.

Interest in the idea of a generic set of skills within and across vocational education arose in the late twentieth century. Various attempts were made by different organisations to define these ‘core’ skills and mathematics began to feature frequently in these definitions. In the 1980s some ‘transferable’ skills, including numeracy, were specified in Youth Training Schemes (YTS) and similar skills were identified in the new BTEC specifications for vocational qualifications. The inclusion of the “Core Skill - Application of Number” in 1992 within the new General National Vocational Qualifications (GNVQ) and National Vocational Qualifications (NVQ) in the 1990s established the concept of an applied form of mathematics as a mandatory component of vocational training[2].

The introduction of ‘key skills’ in the Dearing Report[3] of 1996 led to a redefinition of these generic skills and new forms of assessment (test and portfolio) that were less closely connected to vocational learning and often poorly implemented. Concerns about whether the needs of employers and HE were being met through Key Skills were raised in Professor Adrian Smith’s report of 2004[4] and this led to the replacement of Key Skills by Functional Skills qualifications which centred on application and problem-solving in ‘realistic’ life situations. The development of Level 3 skills was abandoned at this point and Functional Skills qualifications focused on Level 2 and below. The subsequent failure of the Diploma[5] as a vocational qualification and the lack of follow-through of the proposal to couple functional mathematics as part of GCSE left Functional Skills Mathematics as a ‘stand-alone’ qualification. Although suited to contextualised approaches, the connections to vocational learning are not fundamental to the qualification and dependent on localised practice rather than design. The assessment of Functional Skills also highlights the challenge of assessment design in this area.

As evidenced by the relatively quickly changing approach to developing appropriate mathematics for pre-vocational education, and indeed more widely, a successful model has yet to be identified. An approach that allows for separate identification of mathematical competence has the advantage of providing end-users (both students and employers) with the potential for a widely acknowledged mathematics certificate/qualification that has transferability and exchange value. Importantly such qualifications also have the potential to ensure that mathematical competence does not entirely disappear into the vocational practice.

It is in this context that Smith (2017)[6] set the challenge to design mathematics provision that has durability in the new technical education routes. Recent research[7] that investigated mathematics in technical education in six countries (four European and two Pacific Rim) known to have strengths in the education of STEM technicians points out:

> “Changes are iterative, unlike in England where there has been regular whole-scale redesign of technical qualifications. Consequently, in other countries the mathematical demands of technical routes are well-understood and appropriately tailored to the needs of relevant occupations.” (p. 15)

And concludes,

> “…. we consider that there is an urgent need to describe explicitly the mathematics involved in technical occupations. Since there are many synergies in the mathematics across occupations, one potential approach would be to produce a bank of mathematical occupational standards…This would have significant additional benefits in highlighting the relevance of mathematics to young people and raising the status of mathematics for employers and other stakeholders. It would also enable the certification of mathematics in courses of technical preparation, which would be a positive way of promoting greater visibility of, and emphasis on, as well as recognising the value of mathematics in different occupations. This could also support future transferability of mathematical skills and knowledge.” (p.15).

This document provides a rationale for the model of GMCs that seeks to address this challenge.

3 The problem: mathematics education in (and for) the workplace

It is important that the new model of mathematics education for T Levels is cognisant of what is known about the nature of mathematical activity in (and for) the workplace.

There are relatively few research studies that have explored the use of mathematics in workplaces and their contexts. These are often ‘technical’ in nature and perhaps inevitably narrowly focussed. A smaller subset of these studies has sought to develop some form of general understanding so as to potentially inform mathematics education development and design. Here, we set out the main findings from this body of literature which mainly consists of ethnographic studies:

- Workers often do not acknowledge their use of mathematics as it becomes transparent in the routine of workplace activity[8].
- The mathematics that is used is often classified as relatively low level in standard curriculum specifications, but the complexity of its application increases the level of demand of the worker and those seeking to understand the activity[9].
- Mathematics is often applied in technology-rich workplace settings[10].

- Mathematics in workplaces is often ‘obscured’ by workplace conventions which can be idiosyncratic in nature. (A consequence of this is that what is ostensibly the same mathematics might look quite different in different settings).
- Workers often work with/use workplace specific artefacts such as graphical representations that might be understood mathematically but which the worker reads and interprets in terms of the work processes that they represent.
- Use of mathematics is often “black-boxed”[11] in ways that ensure successful outcomes. Consequently, worker engagement with mathematics often only occurs at “breakdown” moments when something needs to be ‘fixed’[12].
- Mathematical models and modelling are used in workplace contexts and the mathematics often needs only be as accurate as necessary for acceptable outcomes (even mistakes or mistaken procedures may be tolerable)[13] .
- Mathematics is used as part of more detailed and complex activity and in this way is part of what in school terms would be considered interdisciplinary activity.
- Mathematics is integral to workplace activity that is socially distributed which involves knowledge exchange between workers both within and across workplace and other communities.

These various research findings were reinforced, to a greater or lesser extent, in the work that ACME produced in 2011 on the mathematical needs of employers[14]. That piece of work added to our understanding through its focus on employers’ needs, for example, reporting that

> “employers emphasised the importance of people having studied mathematics at a higher level than they will actually use. That provides them with the confidence and versatility to use mathematics in the many unfamiliar situations that occur at work.”

This might seem to contradict the earlier point that in workplaces simple mathematics is used in complex situations. However, it is also supportive as it suggests that mathematical preparation for employment would provide workers with the confidence and self-efficacy to tackle problems that arise. In relation to this, it is reported that employers complained that new recruits had, in the main, only learned to do questions that are typically set on GCSE papers and that this had become increasingly inadequate preparation for them to become competent in the workplace. It is perhaps ironic that government policy privileges GCSE retakes when such views are expressed by employers.

The ACME report also underlined the importance attributed by employers to mathematical models and the need for an awareness of mathematical modelling and how underlying assumptions impact on the development and output of those models. This is also evidenced in the research reported above with many descriptive accounts providing insight into a range of workplace specific mathematical models. For example, the work of Hoyles and colleagues[15] described a number of mathematical models in the routines of workers in manufacturing and financial services industries. Elsewhere Wake and Williams[16] found that workers rarely developed mathematical models from scratch but often worked with those developed by others or industry standard models.

Attention is also drawn in the ACME report, as in almost all other research in the area, to the importance of the use of technology/software. This leads to both lower and higher cognitive demand than in the past: often it leads to lower demand in terms of negating the need to apply straight forward routine procedures but to higher demand because of the need for workers to monitor and interpret the output of the technology and software they encounter in relation to their involvement in work processes.

Drawing on twenty-five case studies, in addition to mathematical modelling and the importance of the use of digital technology, the ACME commissioned report highlighted the following aspects of workplace activity as having importance in future curriculum design of mathematics courses that intend to provide for worker preparation:

- costing, including costing of remedial activity in pursuit of successful outcomes, and with integral use of spreadsheets
- working with performance indicators across all aspects of workplace activity
- awareness and understanding of risk
- understanding measures in statistical control and statistical process control.

Further to this, in 2013, Hodgen & Marks[17] in their review of the literature on mathematics in the workplace underlined the prevalence of technology in workplaces and also recommended that

> "To allow students to more easily transfer their mathematical skills into the workplace they should use computers extensively, particularly spreadsheets and computer-generated graphs, to apply and learn mathematics. Competence in these skills matters in the workplace. "(p.4)

In summary, the research evidence on the mathematics of the workplace leads to the conclusion that there is a 'mathematics of school' that to many has become synonymous with the term 'mathematics' itself. The 'mathematics of work' on the other hand, significantly involves the application of this mathematics in ways that involve managing knowledge across the school-workplace boundary. It is the management of mathematical knowledge at/across this boundary that General Mathematical Competences are specifically designed to address.

4 The design challenge from theoretical perspectives

Having explored the context in which GMCs are being developed in terms of policy and research backgrounds, this section sets out the theoretical ideas that underpin their design. At a meta-level this suggests that issues of knowledge management (between the world of mathematics (learning) and vocational settings (application)), models/theories of learning and design for learning need to be addressed.

Firstly, it is important to note that the issue of knowledge management between school and work is often considered from one of two different perspectives, a (information processing) cognitivist perspective[18] or a situated cognition perspective[19]. In general, the former perspective might be considered, perhaps in an over-simplified way, to involve mathematics as an abstract set of knowledge (mathematical concepts, rules and procedures) that can be transferred to, and applied in a range of, new situations. This view, and indeed the transfer metaphor in general, is challenged by the situated cognition view that suggests that all knowledge is contextually situated and dependent and that such transfer is almost impossible. At its extreme this view suggests that (mathematical) knowledge and ideas have to be "relearnt" in each new setting in which one works. Clearly, in developing a workable model for mathematics education for technical vocational routes, it is important to find a workable solution that takes a different approach that both acknowledges the not insubstantial issues that these perspectives highlight and reconciles the not inconsiderable differences.

Knowledge management

So far, the term knowledge management has been used with care here whereas the term 'transfer' may have sufficed and been a more natural choice. However, the workplace mathematics research literature clearly suggests that the transfer metaphor is problematic in relation to mathematics for the workplace. Indeed, researchers such as Beach[20] and Säljö[21] raise this issue and provide alternative conceptualisations and terminology which they use to consider this issue. Importantly, as Tuomi-Gröhn et al[22]. summarise an alternative approach is to focus on knowledge in the changing relationships between individuals and social activities. Beach considers such development as involving horizontal activity as one interacts with the world within a number of different communities, in contrast to the vertical development of knowledge that relates to formal structured educational activity. This suggests that there is a need for an expanded notion of mathematics beyond that of 'school mathematics' in which transformation of mathematical knowledge into workplace contexts might best be facilitated.

Fundamental to designing curriculum that facilitates knowledge transformation activity is recognition and understanding of issues of boundaries, boundary crossing and boundary crossers. Akkerman and Bakker, from a socio-cultural theoretic stance, suggest that

> "All learning involves boundaries. Whether we speak of learning as the change from novice to expert in a particular domain or as the development from legitimate peripheral participation to being a full member of a particular community (Lave &

Wenger, 1991)[23], the boundary of the domain or community is constitutive of what counts as expertise or as central participation."[24]

The implication is that careful consideration needs to be given to the boundary between learning mathematics in educational contexts and its use in the workplace, and most importantly the relationship between the nature of mathematical knowledge across such boundaries

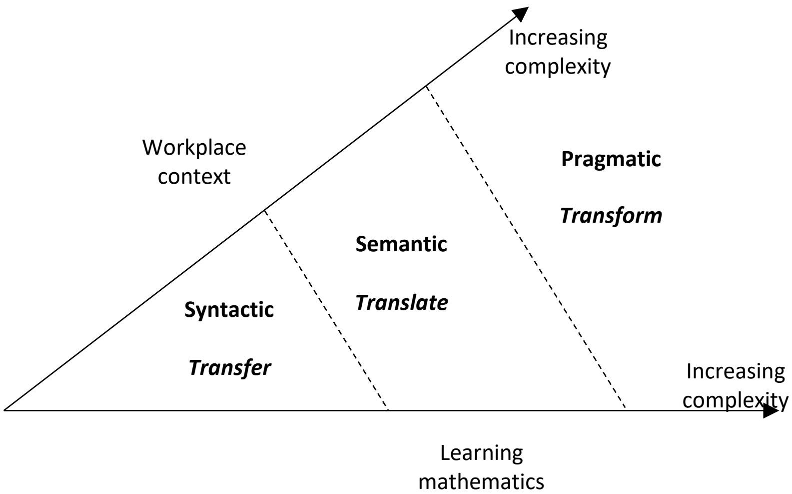


Figure 1: Carlile's Integrated/3-T Framework for Managing Knowledge Across Boundaries

In the organisation science literature, Carlile[25] proposes an integrative framework to conceptualise knowledge management across boundaries. This describes three progressively complex boundaries—syntactic, semantic, and pragmatic and three progressively complex processes that can be used to consider how knowledge might be managed in each: transfer, translation, and transformation (see Figure 1). This conceptualization suggests that knowledge transfer can be relatively unproblematic when one is faced with a syntactic boundary, that is when there is a common lexicon across a boundary which acts to support knowledge *transfer*. Perhaps the use of mathematics in other school subjects could be considered to be of this type, although even in such settings transfer of mathematics is known not to be without its issues when even slight changes in syntax occur (for example, the term 'gradient' in mathematics might be replaced with 'slope' by (some) science teachers).

However, more problematic is when the boundary requires novel aspects in terms of the use of knowledge in the new setting resulting in differences and ambiguities in the understanding of knowledge in the different settings. In such circumstances the boundary is considered to be semantic and there is a need for knowledge *translation* which requires active work to develop shared meanings within and between each setting.

Pragmatic boundaries occur in more complex situations involving management of knowledge across boundaries. In such circumstances it is suggested that there are important differences among the actors on either side of the boundary. This seems to be the most appropriate situation when considering the learning and use of mathematics in the two distinct settings of school and workplace. In workplaces, workers focus very clearly on workplace outcomes in terms of production or services and in such circumstances mathematics is applied as being in the service of, and subservient to, success in these terms. At such pragmatic boundaries, Carlile suggests that

> "actors must be able to represent current and more novel forms of knowledge, learn about their consequences, and transform their domain specific knowledge accordingly. This knowledge is a *transformed* [our italics] mixture of the knowledge determined to still be of value and the knowledge that has been determined of consequence given the novelty present."[26]

The school-workplace boundary is of this type with new participants in workplace activity being required to transform the relatively pure and abstract knowledge of 'school mathematics' in ways that are both novel and situated both in general pathway-specific terms as well as in ways that might be very much focused on a particular employment setting. For example, Wake[27] reports how college students, as part of a research project researching the use of college mathematics in workplace settings, had difficulties in bringing together the concepts associated with the school mathematics of average and gradient, to understand the calculation of average gradient in the workplace context of railway signal engineering. In the workplace context of a railway signal engineer the mathematical activity not only brings together two key mathematical concepts it also requires understanding of gradient where the values being used are both relatively very large (run) and small (rise) requiring attention to issues of error and tolerance.

Learning

Theories of learning abound with some being quite narrow in their conceptualisation of learning, focussing in the main on the cognitive/conceptual understanding of the individual[28] whereas others take a wider view and consider social aspects and development of a learner's identity[29], for example. Such a broader understanding of learning seems particularly helpful in the case of learning in and for the workplace as the learner is required to develop a new relationship with both mathematics as a knowledge domain and workplace activity which presents an equally demanding changing relationship with socially situated activity. As identified above, mathematics as knowledge needs to be transformed into new settings and situations: transformed in ways that allows the learner to become a participant in a workplace community of practice (in Wenger's[30] terms). Wenger puts forward a social theory of learning that develops from his construct of 'communities of practice'. He defines a community of practice as a group of people who develop a common activity and who interact regularly to learn how to do it better in ways that support their mutual engagement, joint enterprise, and shared repertoire. In workplaces such communities focus on production or service and support workers as learners in developing their practice, their identity as a certain type of worker, their making of meaning of their activity in terms of their work and their engagement in the workplace community. Importantly, this learning not only refers to their practice (*doing*) but also their sense of *becoming* a worker. In the new Technical Level qualifications this will involve what might be considered a period of induction that aims to support students' understanding of what it means to become a future worker in a particular vocational pathway. Consequently, an appropriate mathematics curriculum needs to be such that put simply captures to some extent what it means to behave and work mathematically in the vocational context. The formulation of the curriculum must be such that as well as engaging with the content of mathematics it also provides opportunities to provide a sense of becoming a user of mathematics in ways that are appropriate to the particular vocational route and future workplace and to have experience of using mathematics to make meaning of worker/workplace activity and to have a sense how this supports a community of practice.

The implication for curriculum design of mathematics for T Levels is that the mathematical experience of students needs to be significantly different to that of 'school mathematics' and should facilitate students' experience of mathematics, at a minimum, in quasi-workplace scenarios.

Boundary objects

Sociocultural approaches to learning emphasise the importance of boundaries in the sense of sociocultural differences and discontinuities between the different activities in which learners might engage. For example, the boundaries inherent between learning mathematics and workplace activity. The important focus for learners on courses leading to T Levels is their engagement with knowledge that facilitates workplace competence rather than the learning of mathematics for its own sake: ideally their mathematics learning will have obvious *utility and purpose* (in Ainley & Pratt's [31] terms) for future workplace competence.

In sociocultural theoretic terms, engagement in different activity systems or communities, such as those focussed on school/college and work, requires boundary crossing and this can be facilitated by boundary objects. Boundary objects are artefacts that are structured in ways that they have global meaning in general across the boundary but are also sufficiently 'plastic' to be locally adaptable[32]. For example, a table of ingredients or recipe can be used by a cook/ catering manager as a 'shopping list' when scaling up to order supplies and can be used by a mathematics teacher to teach the mathematical concepts of proportionality, ratio, etc.

To design a mathematics curriculum that supports both learning of mathematics and vocational competence there is a need to design boundary objects that have meaning to teachers and learners of mathematics and to teachers, learners and practitioners in the vocational area. Experience suggests that often the design and structuring of mathematics curricula prioritises almost exclusively what we have referred to as ‘school mathematics’ and its teaching and learning rather than considering how best to facilitate knowledge exchange, indeed knowledge transformation, across what is effectively a complex boundary.

5 The design challenge

In summary, then, the challenge for curriculum design and subsequent development of mathematics in ways that are appropriate to T Levels is to design a curriculum that as a boundary object speaks in meaningful ways to teachers and learners of mathematics to facilitate knowledge transformation across the mathematics/vocational boundary. The analysis herein suggests that this design needs to:

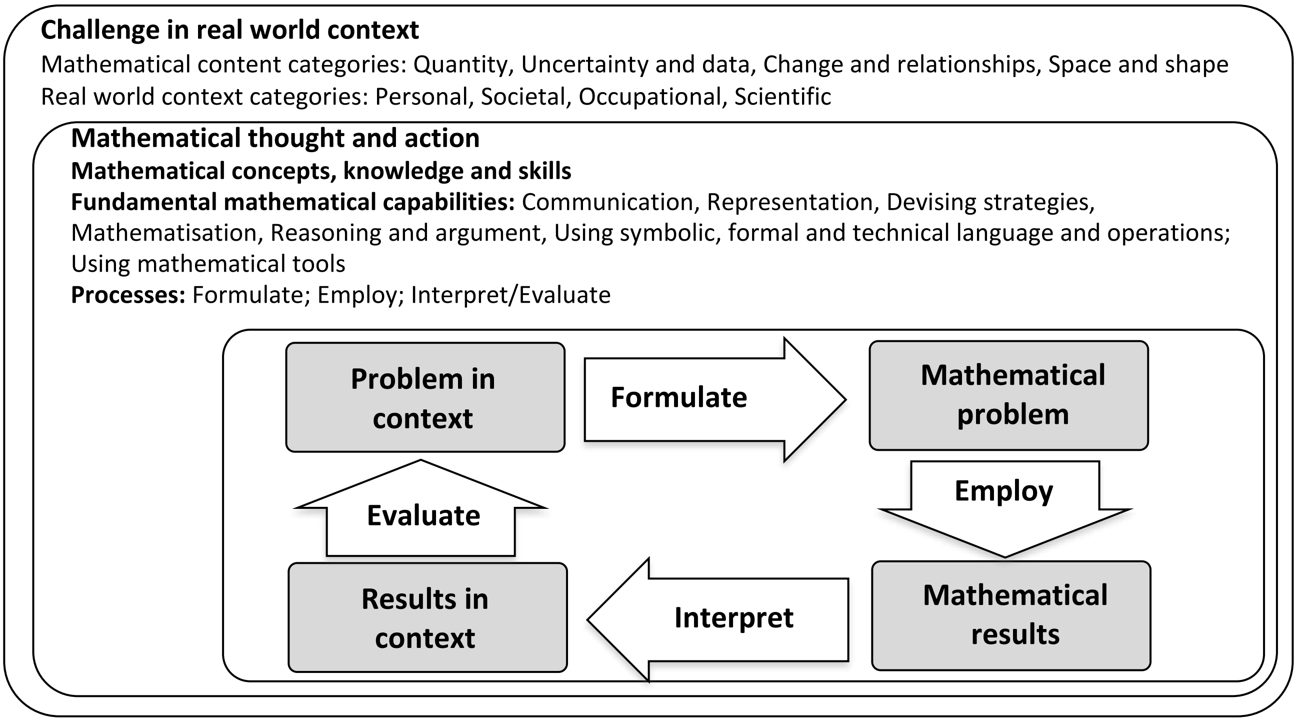
- Ensure that the learners have awareness of how mathematics is developed and applied in a vocational domain in ways that are potentially idiosyncratic but which are informed so as to be consistent with mathematics more generally;
- Acknowledge that simple mathematics is often used in complex ways
- Provide learners with a sense of developing mathematical practice that has meaningful utility and purpose;
- Supports identity development for those entering a vocational community in which mathematics informs wider work practices.

Ideally, this curriculum will also provide a common structure around which mathematics for all T Levels can be specified whilst also facilitating adaptation that ensures authenticity of student experience in each.

6 The mathematics

Mathematics as a discipline or domain of study is quite rightly a contested area. For many, the term ‘mathematics’ conjures up ‘school mathematics’ as a list of content, often organised around four main content areas: number, algebra, geometry/shape and probability/statistics. Curricula are often specified under these main headings and with the detail arranged into lists that in some way signal increasing complexity in terms of conceptual understanding informed by both empirical research[33] and classroom-based observation. In addition, curricula invariably take account of mathematical practices or process skills in one way or another to signal what the designers think it means to be mathematical or work with mathematics. This has led to a proliferation of terms that are used to describe such curriculum formulations: numeracy, mathematical literacy, quantitative literacy, functional mathematics, and so on. These curricula may be more or less restricted in nature in terms of their focus on number and quantitative aspects of mathematics, rather than encompassing the other content sub-domains identified above (algebra, geometry/shape and probability/statistics). Also ‘big ideas’ or ‘ways of being mathematical’ are often referred to in statements that set out overarching curriculum objectives in school mathematics and these may permeate through to assessment objectives. For example, mathematical reasoning, problem solving, modelling and so on may provide foci for aspects of the curriculum and the intention may be to encourage ‘ways of being mathematical’ that are consistent with such ‘big ideas’.

A number of different frameworks that attempt to identify what it takes to be effective in applying mathematics have gained international recognition in the literature, and indeed in informing the development of both international testing and curriculum specification development in many countries. Pre-eminent of these is the OECD’s PISA framework for mathematics[34] that is updated and developed from time-to-time in line with the cycle of testing. Fundamental, to the development of the framework is the notion that mathematics is brought together to solve problems in different *contexts* in ways that requires the learner to work with mathematical *concepts*, drawing on a number of mathematical *competencies*. The diagram of Figure 2 provides a schematic overview of PISA’s “mathematical literacy” knowledge domain. This is summarised as:



> "Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens."[35]

In the PISA formulation of mathematical literacy, a modelling perspective is central: the modelling cycle clearly identifiable in Figure 2 which highlights the key process skills of formulating, employing, interpreting and evaluating (that is, the processes involved when working between mathematics and a contextual situation).

A further helpful conceptualisation of mathematical activity is that informing the design of international testing of adults' numeracy skills in the OECD's PIAAC programme[36]. In this case the

Figure 2: "A model of mathematical literacy in practice"[37]

working definition of numeracy that brings together the skills and competencies required by adults to cope with tasks that are likely to appear in the adult world and that contain mathematical or quantitative information is:

> "Numeracy is the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life."[38]

Recognising the importance of applying knowledge the PIAAC conceptualisation of the domain identifies what it means to exhibit *numerate behaviour*: "Numerate behaviour involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways. "

This is expanded in Figure 3 below[39].

Numerate behaviour involves managing a situation or solving a problem….

1. In a real context:
   - everyday life
   - work
   - societal
   - further learning
2. By responding:
   - identify, locate or access
   - act upon, use: order, count, estimate, compute, measure, model
   - interpret
   - evaluate/analyse
   - communicate
3. To mathematical content/information/ideas:
   - quantity & number
   - dimension & shape
   - pattern relationships, change
   - data & chance
4. Represented in multiple ways:
   - objects and pictures
   - numbers & mathematical symbols
   - formulae
   - diagrams & maps, graphs, tables
   - texts
   - technology-based displays

Numerate behaviour is founded on the activation of several enabling factors and processes:

- Mathematical knowledge and conceptual understanding
- Adaptive reasoning and mathematical literacy skills
- Beliefs and attitudes
- Numeracy-related practices and experience
- Problem-solving skills
- Context/world knowledge

Figure 3: PIAAC's model of mathematical literacy in practice

The development of these frameworks has been informed extensively by the work of Niss and colleagues in Denmark who have, over many years[40], considered how a number of fundamental mathematical capabilities, or mathematical competencies, are drawn upon when engaging with (mathematical) problems. Niss and Jensen[41] (2002) write:

> "*mathematical competence* means to have knowledge about, to understand, to exercise, to apply, and to relate to and judge mathematics and mathematical activity in a multitude of contexts that actually do involve, or potentially might involve, mathematics." (p.43)

They identify eight mathematical competencies:

- Mathematical thinking competency
- Problem handling competency
- Representations competency
- Modelling competency
- Reasoning competency
- Symbols and formalism competency
- Communications competency, and
- Aids and tools competency.[42]

Niss and colleagues draw a fundamental distinction between mathematical literacy and mathematical competence/competencies by suggesting that the former is concerned with aspects of activity focussed on the everyday societal aspects of life with mathematical competencies being concerned with the application of mathematics more widely than this.

Other researchers have developed different models of mathematical literacy which attempt to capture the essence, and complexity, of what is involved. For example, Goos and colleagues[43] in their schematic overview (Figure 3) position *contexts* as central to their model identifying these to potentially lie in areas of citizenship, work, or the personal and social. Mathematically literate activity is then identified as being reliant on mathematical knowledge, tools and dispositions.

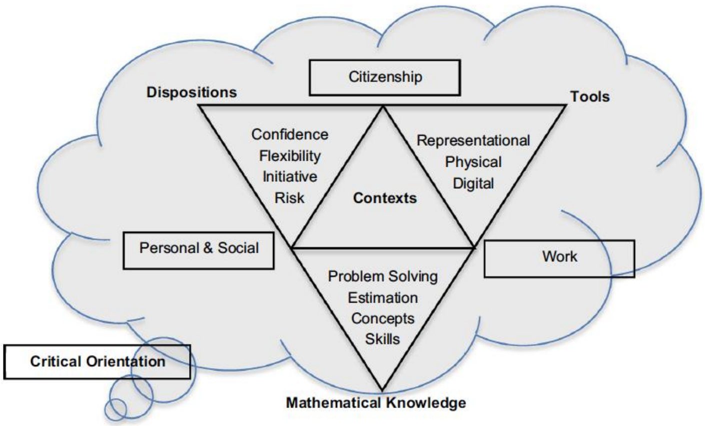


Figure 3: Goos and colleagues' model of mathematical literacy[44]

In this model, Goos and colleagues expand ideas of mathematical knowledge to include problem solving and estimating strategies in addition to concepts and skills. In doing so they recognise the need for a creative approach and self-monitoring of progress made when solving problems in ways that require considerably more than the straightforward application of rules and procedures.

This brief discussion of the work that has considered aspects of applying mathematics to solve meaningful problems, primarily situated in a world outside of mathematics itself, identifies the complexity of such problem-solving activity and provide a number of different theoretical conceptualisations that provide insight into contributory factors. The question remains as to how best take account of such factors whilst narrowing the focus to that of (mathematical) problem solving in and for the world of work.

7 Digital technology

The importance of the use of digital technology in workplaces and how this can interact with mathematical activity has been referred to above. This requires considerable attention in relation to the development of mathematics in T Levels to ensure that students are well-prepared for the changed and additional demands that the programme should make in this regard.

Hoyles and colleagues in the Executive Summary of their report, Mathematics Skills in the Workplace[45] wrote,

> "All the sectors exhibit the ubiquitous use of Information Technology. This has *changed the nature of the mathematical skills required, while not reducing the need for mathematics.* On the contrary, in many cases, a competitive and IT-dependent environment means that many employees are using mathematics skills that their predecessors, or they themselves in the past, did not require." (p. 10)

This report was written to the Science, Technology and Mathematics Council as long ago as June 2002. Since then, of course, the role of digital technology in workplaces has only increased, and at pace. In later work Hoyles' research group coined the term techno-mathematical literacies[46] to emphasise the important role that digital technology has to play for those becoming mathematically literate in workplace settings. Indeed, their action research focused on designing TEBOs (technology-enhanced boundary objects) which used technology to provide employees opportunities to explore the mathematics of the models used in their work and how these impact on work processes and outcomes. For example, a TEBO may allow one to explore how output is affected by varying different parameters in the workplace process. Designing and developing TEBOs bespoke to the work of particular professionals allowed the researchers to explore aspects of a new mathematics for the workplace as opposed to school mathematics.

More recently the ACME Report of 2011[47] concluded: “The use of computers is universal in the workplace, and, as a direct consequence, the demand for mathematical skills has increased.” (p.25). However, school mathematics rarely pays attention to this issue in a meaningful way and because of this there is, as van der Wal and colleagues[48] suggest, a “discrepancy between school and work mathematics” and consequently “students are insufficiently prepared for their future jobs”.

It is essential, therefore, that the mathematics in T Levels is developed in ways that are sensitive to this issue. Not only should students gain experience of working with mathematics in ways that reflect how it can be transformed by digital technology, they should also have hands-on experience of using appropriate industry-standard software where possible. A particular case is that of widely used spreadsheet software, often Excel, as this is used almost universally across many different workplace settings in many situations. It is important that all students have experience of working with this software and that it replaces other classroom-based technologies often used in the teaching and learning of mathematics such as graphic calculators, which whilst being efficient in focussing teaching and learning in ways that provide insight into the mathematics, diverge considerably from how mathematics is used and experienced in workplace settings. It is important that, in T Levels, mathematics is experienced using digital technology in ways that better reflect workplace experiences.

8 Proposed solution: General Mathematical Competences

The construct ‘General Mathematical Competence’ (GMC)[49] was devised as a result of analysis of the mathematical curriculum required by science students following prevocational courses. The resulting outcomes arose from asking not only the question, ‘What mathematics do pre-vocational science students use?’ but also, ‘How do pre-vocational science students organise their use of mathematics?’

The result was an initial seven GMCs (since expanded to ten) which cover a wide range of mathematical content and skills that were required in meeting the demands of prevocational science courses. Each GMC:

- applies in many situations and is sufficiently general to be useful;
- organises a substantial body of mathematical knowledge (concepts, skills and understanding);
- emphasises the use of mathematical models that provide mathematical insight into real (and complex) situations and problems that are simplified by making a number of assumptions.

The construct of General Mathematical Competence importantly provides a boundary object between the world of work and education in that it clearly has meaning in each community, albeit with a very different emphasis in each. For example, in the world of work ‘Costing a project’ relates to identifying costs in terms of money, components, time, employees, and so on in pursuit of a work-related objective whereas to the mathematics educator the activity focuses on the detailed accurate calculations required. Consequently, the learner’s ‘costing a project’ activity provides for multiple meaningful experiences including learning how to apply mathematics with workplace utility and purpose and providing for the development of their identity as a future worker in a particular vocational area.

Each general mathematical competence stands independent of the ‘level’ of the mathematics being used. Hence, level 2 and 3 technicians as well as graduate technicians might, for example, all be ‘costing a project’ but the complexity of this costing could range from ‘simple’ to ‘difficult’ both in terms of the number, range and type of factors involved and consequently the difficulty of mathematical working required.

The following outlines a framework of ten General Mathematical Competences:

- Measuring with precision
- Estimating, calculating and error spotting
- Working with proportion
- Using rules and formulae
- Processing data
- Understanding data and risk
- Interpreting and representing with mathematical diagrams
- Communicating using mathematics
- Costing a project
- Optimising work processes

9 GMC descriptors

Measuring with precision

In many workplace situations measurement is important. Due attention should be paid to the use to which the raw and processed data will be put as this will inform what should be measured as well as the required *accuracy* and units of measurement.

Cumulative errors are understood as is the effect that errors in measurement have on subsequent use of values in further processing (such as when used in formulae), and the accuracy, or precision, required for a particular purpose. Upper and lower bounds are considered when appropriate.

This *GMC* includes the critical interpretation of calculator or spreadsheet displays [e.g. understanding calculator "rounding errors" such as $2/3 \times 3 = 1.99999998$).

Ideas of absolute and percentage errors are considered and understood

Issues concerning the calibration of instruments are covered in this *GMC.* This *GMC* has strong links with other *GMCs*.

Estimating, calculating and error checking

People with this GMC can apply routine skills with confidence and fluency to solve technical problems. They use their knowledge of context to find appropriate and approximate solutions to calculations. In particular, estimations, perhaps using rough rules of thumb, may provide a starting point to 'get a sense of', whereas more accurate calculations may be required at a later stage. They might use specialised notation/representation reflecting industry standard practice, and depending upon specialisation, be particularly good at understanding how to work with very large and/or very small numbers.

As well as understanding estimation, sources of error and approximation they can recognise the impact of these on the accuracy of solutions to problems in applied contexts.

Working with proportion

The use of proportion permeates many areas of mathematical application (e.g. other *GMC*s require the scaling of data which requires an understanding of the concept of proportionality).

This *GMC* requires numerical, graphical and forms of algebraic understanding of this key mathematical concept so that one can move with ease between different understandings and recognise situations where ideas of direct proportionality and inverse proportion apply (and those where they do not).

Users should be able to apply proportional reasoning with fluency to solve problems and model situations. They should be able to generalise proportional thinking in words, forms of algebra/formulae appropriate to vocational specialisms.

Using rules and formulae

This *GMC* is concerned with the use of rules, algorithms and formulae. It recognises that formulae are represented in different ways in vocational settings including as basic rules of thumb (maybe passed by word of mouth), in words or via numeric ‘ready reckoners’, in ‘spreadsheet algebra’ or in formal algebraic representations.

There is often a need to use formulae to process data. Selecting appropriate data and paying attention to units/dimensions of quantities is important. It might be necessary to manipulate or ‘rearrange’ formulae, using them to find solutions to problems and/or interpret outcomes in terms of the original problem.
Having a sense of what may or may not be correct is important when using formulae to find solutions to problems.

‘Real world’ formulae are often presented differently from those typically met in school although also conveying important ideas of structure and relationships of a situation/context. They may have been amended by different people using industry standard or idiosyncratic conventions and/or notation. An important skill, therefore, is to be able to understand changes and identify and/or remove redundant information or symbols.

Processing data

This *GMC* includes using technology as appropriate to carry out the systematic collection, processing and organisation of data into usable forms (e.g. tabular or graphical) in preparation for reporting and/or interpretation.

Depending upon the context, this might include 1) the identification of suitable data, 2) its collection or generation 3) systematic organisation and recording prior to any scaling or processing that may be required. Graphing will require use of software to scale axes (etc.) to plot the raw or processed data and produce diagrams, graphs and charts that best communicate information to intended audiences and reflect ‘industry standard’ practice.

Generating and/or interpreting graphical outputs, whether developed by oneself or others (including those produced automatically by technology), requires attention to the above details and mathematical reasoning.

Understanding data

Access to primary and secondary data is increasingly common in a range of settings and often using appropriate technology. This *GMC* requires critical understanding and interrogation of how such data may be summarised in graphs, informatics and summary measures, and used to make predictions.

Understanding how data are generated, sifted, selected (sampled), organised is required together with understanding of bias and approaches to interrogation.

Attention to how the data have been processed, scaled and how they are presented using summary measures of location and spread and by a range of visual/graphical display is required.

Understanding graphical data may require the identification and validation (using technology) of mathematical functions to appropriately model the data.

Probability should be used to calculate appropriate uncertainty in making predictions and exploring future risks.

Data processing procedures and outputs should be interrogated and interpreted critically.

Representing with mathematical diagrams

People with this *GMC* are able to translate situations into appropriate diagrams and representations that highlight key data. They can also interpret such diagrams and representations. Technology is used that is appropriate to the task and this *GMC* includes working with plans/scale drawings, maps, linear programming graphs and other diagrams which may be particular to one or more employment routes.

The *GMC* includes the ability to select salient features of contexts, their scaling/processing and representation, and the reverse of this process. Industry specific conventions and notation should be applied. Technology should be used that is appropriate to the task.

Working towards a final output/product will require ongoing interpretation and checking of working and results. This will involve interpreting back and forth between representation and reality. It is likely that a range of different representations of the same situation may be developed/used depending on the intended audience and level of detail and accuracy required.

Communicating using mathematics

People with this GMC are able to use mathematical processes (calculations, diagrams and data representations) to support technical arguments and communicate effectively to a range of stakeholders. They can reason with mathematics, communicate this clearly and draw conclusions that are persuasive within the context of the problem situation.

Communicating with mathematics in this way will require the use of other GMCs. However, how outcomes of working are presented will be informed by the intended purpose and audience of the communication. This will require use of industry standard conventions/notation and sensitivity to the likely background and knowledge of the intended audience.

Costing a project

This *GMC* (which might also include other *GMCs*) requires the calculation of the ‘cost’ of a (substantial) project or activity. The costs in question may be financial, but could also be in terms of, or include, quantities of a product, use of space, amount of labour and so on. The scope of the activity could vary considerably from costing quantities of components required for a day’s work away from the depot through to costing the total hours required by different workers on a large-scale construction project.

Activity will include:

- identification of all relevant factors / components
- calculation of the cost of individual parts of the project
- compounding of these individual components to give an overall cost.

This *GMC* would include systematic organisation/tabulation of calculations and results and presentation of these, for different audiences and at different levels of detail, using tabular and graphical/visual means.

More complex activity may require experimentation using the developed model for costing to assist decision making (for example, when attempting to meet, reason and justify a budget) and/or consideration of probabilities/risk associated with various aspects of the project to provide insight into potential variation in costing

Optimising work processes

This *GMC* focuses on modelling in order to organise and optimise a substantial or complex piece of work. For example: ensuring building materials are available at the correct time and place in a construction project; developing a website by bringing together modules of coding; ensuring workers are available according to anticipated demands, and so on.

An organising structure of the modelling cycle may inform activity. This will involve:

- having a thorough understanding of the context of the work
- identifying key factors that will be taken into account
- making assumptions including considering probabilities, risks and so on
- developing a solution*
- interpreting and considering the validity of the solution in light of the context of the work
- communicating a fit-for-purpose solution in an appropriate format.

* developing a solution is dependent on the context of the work/project/programme being organised. For example, it may involve developing a critical path analysis when organising an event, it could be the development of coding for part of a website or computer application and so on. A range of possible solutions may be considered taking into account various probabilities/risks associated with aspects of the work if appropriate.

10 Implementation of GMCs in T Levels

Fundamental design principles of the GMCs:

Each GMC identifies a substantive and coherent aspect of workplace activity and depending on the particular context intersects with and work in synergy with other GMCs. It is likely, therefore, that when engaged with mathematics within workplace contexts this will involve more than one GMC. For example, in the case of costing a particular project it may be necessary to draw on mathematics associated with estimating, calculating and error spotting and working with proportion whilst also drawing on outcomes associated with optimising the work processes involved so that the cost is potentially minimised. The starting point in ensuring the effective application of mathematics in work towards a T Level is to identify vocationally related activity and then determine which and how GMCs contribute to and enhance this. A number of examples of this approach are exemplified below.

T Level – Construction: Design, surveying and planning
Context: Energy Performance Certificates and sustainability

Although the calculations associated with providing information for Energy Performance Certificates are complex and rely on a wide range of factors including for example, the heating system(s) in place, roof construction and so on it might be expected that students would have a sense of how different factors contribute to energy efficiency. For example, students may be expected to understand the impact of using different materials in the construction of a building. To do so they may explore the impact on heat loss of changing from single glazing to double glazing in building construction by calculating the percentage improvement in heat loss through a window of a given size when glazed by different types of window.

This requires working with the scientific model for heat flow:
The heat transmission, *H*, through a material can be expressed as: $H = U\,A\,\Delta T$ where *H* the heat flow is expressed in Watts (or joules per second), *U* is the overall heat transfer coefficient expressed in $Wm^{-2}K^{-1}$, *A* m^2 is the area of the material and ΔT is the temperature difference with temperature measured in degrees Celsius (or Kelvin).

U values for various models of glazing can be found. For example:

Glazing type	*U* value ($Wm^{-2}K^{-1}$)
Vertical single glazed window in metal frame	5.8
Vertical single glazed window in wooden frame	4.7
Vertical double glazed window, distance between glasses 30 - 60 mm	2.8
Vertical sealed double glazed window, distance between glasses 20 mm	3.0
Vertical double glazed window with "Low-E" coatings and heavy gas filling	1.5

This could be extended to calculating *U* values for compound materials composed of different layers of materials for which the *U* values are known. This requires working to determine *U* where $U = \frac{1}{\sum R}$ where *R* is a material's resistance to heat flow measured in m^2KW^{-1} and $R = \frac{1}{U}$.

GMCs
In this case students will be most clearly engaged in activity appropriate to the GMC *Using rules and formulae*. They will need to understand how to calculate heat transmission using the scientific model/formula using values that are industry standard paying particular care to attend to units (as values given may be typical rather than specific and may at times be given in units that require converting into those consistent with the formula).

Given the nature of the formula attention can be directed to the GMC *Working with proportion* by considering how heat flow varies as the *U* value varies keeping all other factors constant. Exploration of this may involve developing spreadsheet calculations and considering the connections between algebraic formula, spreadsheet formulae and graphical representations of this proportional relationship. If extending explorations to *U* values of compound/composite materials it will be necessary to consider *U* and *R* as reciprocals of each other potentially leading to experience of inverse proportionality. Work in this context also provides for experience of engagement in *Estimating, calculating and error checking*.

Indicative content	
Calculating with decimal numbers Unit conversions and compound units Dimensional analysis Considering accuracy Direct and inverse proportion	Approximating to check validity of calculations Using formulae Ideas of functions, graphs, rates of change

T Level – Legal, Financial and Accounting

Context: Business finance – stocks, shares and financial indices.

It would be expected that in this T Level students would have a general understanding of industry standard models of communication such as charts and indices that are used to convey quickly information relevant to a range of users. Important in this regard are financial charts known as candlestick charts that can be used to illustrate price movements of a range of financial products such as shares, as well as trading in currencies, and stock market indices amongst others.

The candlestick chart below, for example, shows the change in the FTSE 100 index week by week over the period of one year (although traders tend to use such charts over shorter timespans with a candle often being used for each day's trading).

Interpretation of candlestick charts together with an understanding of the principles behind developing an index such as the FTSE 100 is an important aspect of financial literacy in a business context.

GMCs

When interpreting charts of financial information such as in the case of candlestick charts students have to learn industry specific conventions and also how to make sense of the data mathematically. Candlestick charts in particular provide immediate insight into both short-term detail as well as longer term trends. This provides opportunities for engagement in the GMC *Representing with mathematical diagrams*. As in the case of the diagram above there is the opportunity to 'read' the candlestick chart alongside the accompanying bar chart and details such as the scaling of the vertical axis needs to be taken into consideration.

Although it would not be expected for students to have to be able to calculate values of the FTSE 100 index it is important that they have an understanding of how such an index is developed possibly using a small number of constituents to develop an index by weighting the contributions of each constituent part. Such understanding will involve working with the GMC *Using rules and formulae*.

Indicative content	
Calculating with decimal numbers and percentages Considering accuracy Scaling / proportional reasoning	Principles underpinning calculation of an index in relation to a baseline Visualising/ communicating data using non-standard mathematical representations

T Level – Education and childcare
Context: Calculating a fees break-even point

As in many small business contexts, it is important that when considering the viability of offering childcare services that some exploratory calculations are undertaken. For example, calculations might be carried out to determine a possible hourly rate.

This might involve the following steps:

- Calculating total annual expenditure. (At this point considering only fixed costs that are independent of the number of children being cared for);
- Calculating weekly expenditure (dividing the annual expenditure by the number of weeks for which childcare offered);
- Calculating the number of hours available each week (multiplying the number of hours of childcare offered by the number of registered places);
- Using a likely occupancy rate to find the number of hours likely to be sold each week;
- Finding the break-even hourly rate by dividing the weekly expenditure by the number of hours likely to be sold each week.

It is likely that such calculations would be carried out using a spreadsheet so that assumptions might be varied and graphical outputs could be used to compare and contrast different scenarios.

Such relatively simple costings might be developed further by considering issues of profit and loss, cash flow and so on. It may be necessary to develop diagrammatic output to help communicate the information to a potential business partner or maybe a bank if setting up a business account. In this context it may also be helpful to undertake some probability risk calculations to take into account some potential variation in occupancy rates.

GMCs

The activity identified here provides an example of how students may be involved in *Costing a project*. This will require considerable understanding of the workplace context: in this case, for example, the costs associated with offering childcare including costs associated with registration (annually), any modifications to accommodation, heating, lighting and so on. As suggested above it is likely to be advisable to carry out the necessary calculations using a spreadsheet so that factors/assumptions can be varied and explored. Depending on the approach taken it may be that students also engage in mathematical activity contributing to the GMCs

Using rules and formulae: working with formulae in words (as above) and translating these into spreadsheet algebra

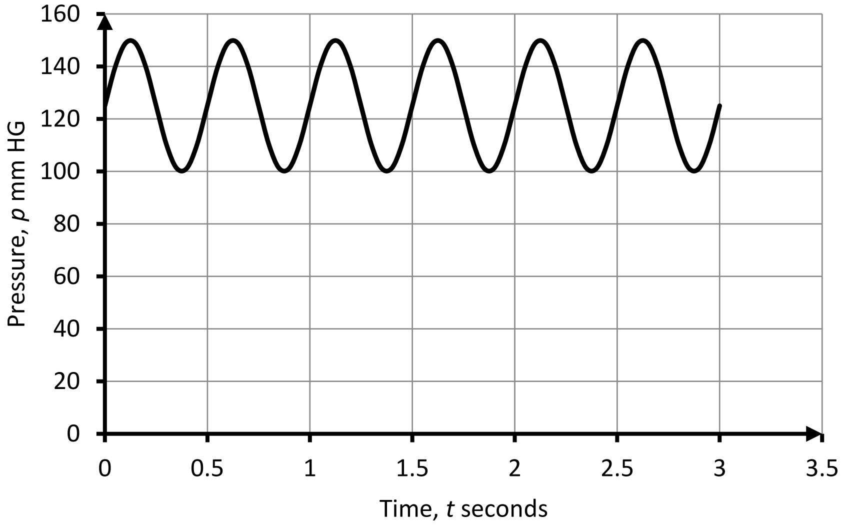
Communicating using mathematics: providing a financial case/argument (potentially for a business partner or bank) that supports a proposed pricing structure.

Indicative content		
Carrying out complex calculations with decimal numbers, fractions, percentages, money Considering units such as cost per hour.	Working with formulae expressed in words, using spreadsheets Communicating mathematical reasoning and outcomes of calculations including with mathematical diagrams	

T Level – Health and science

Context: Understanding blood pressure measurements

In many health settings a standard procedure is for patients to have their blood pressure monitored. This is often done quite simply using a small electronic device and a cuff that is attached to the upper arm. The device inflates the cuff and measures the person's blood pressure as it deflates. It gives at least three important readings: the systolic (maximum) pressure, the diastolic (minimum) pressure and pulse rate with the pressure readings given in millimetres of mercury (mm Hg). To support students' understanding of the underpinning scientific principles they will have to engage with, and understand, graphs that show how blood pressure varies with time (as in the diagram below which shows the blood pressure of someone who has been exercising and who has a pulse rate of 120 b.p.m. and a blood pressure of 150 over 100).



It may be that this is just one of a number of charts and graphs that students will be expected to understand and interpret. For example, this may be considered in conjunction with other graphs and data that refer to respiration and electrocardiogram traces.

GMCs

Understanding data and the mathematical diagrams referred to in this context will involve students in working within the GMC *Representing with mathematical diagrams*. In particular students will have to select salient features of the diagrams (for example, in relation to the graph shown above) and understand these in terms of the functioning of the heart, breathing and so on. This will require close attention to details of scaling and understanding how the period of the graph can be determined and its meaning etc. Issues of scaling and calibration of instruments etc. is also likely to involve students in working within the GMC *Measuring with Precision.*

Indicative content	
Unit conversions and compound units Considering accuracy and precision	Approximating to check validity of calculations Using formulae Ideas of functions, graphs, rates of change

T Level – Construction

Context: Project management

Alongside the work required to cost a construction project it is essential to consider project planning to ensure that work can proceed through a logical sequence of activities over the period of the construction project. For example, students might engage in considering the planning of a household extension such as a new bathroom. It is important in such a case to proceed from initially constructing the shell of the building that will house the bathroom, through each stage until the final decoration of the room can be completed. Important aspects that need to be considered include ensuring that

- workmen are available at the right time to be able to work on the different phases of the construction;
- specialist construction equipment is available when required
- materials are available when needed and are available in the right quantities.

The outcomes of such planning need to be communicated to a range of others both at the outset of the work and as it proceeds using a range of mathematical diagrams.

GMCs

Project management of the type referred to here sits squarely within the GMC *Optimising work processes*. Fundamental to such planning is a thorough understanding of the workplace context, for example, being aware of and sensitive to how different aspects of the work depend on others having been completed or how they may have to be developed in parallel for successful completion (for example, if fitting an electric power shower, it may be necessary to have both a plumber and electrician available on the same day. In completion of this work it may be necessary to engage with a number of different types of mathematical diagrams that facilitate both mathematical/logical thinking and communicating with others. For example, a Gantt chart may be used to organise daily activity, whereas a simpler bar chart might be used to communicate plans to workers. It may be necessary to consider aspects of probability/risk relating to times required for various aspects of the work and use these to inform variation in potential plans.

In developing the optimal work plan it is likely that students will also engage with aspects of the GMCs *Representing with mathematical diagrams* and *Estimating, calculating and error checking* as it in many instances it is likely that approximate solutions to underpinning calculations will suffice.

Indicative content	
Carrying out complex calculations with decimal numbers, fractions, percentages, money Considering compound units such as cost per hour. Approximating to check validity of calculations	Working with formulae expressed in words, using spreadsheets Communicating mathematical reasoning and outcomes of calculations including with mathematical diagrams Scaling, proportionality

11 Assessment

The assessment of mathematics in T Levels requires careful consideration. It is not the intention here to promote any particular model of assessment but rather to raise important issues that should be considered in assessment design. These will help to ensure that the principles and intentions of the GMCs are realized and not undermined by the assessment process.

The GMCs are designed to enable learning of mathematics at the boundaries between the worlds of school and work. This innovative approach to curriculum design is dependent on there being appropriate, well-designed assessment. Fundamentally, the assessment model adopted needs to ensure curriculum fidelity[50] requiring that the curriculum and assessment are well specified and that the assessment ensures broad coverage of the curriculum, if not of each specific sub-domain. For this reason, it is important to align instruction, learning and assessment[51] so that each is mutually supportive of the others. This is a considerable challenge as many traditional assessment practices tend to focus on assessment *of* learning rather than assessment *for* learning in ways that drive teaching for assessment instead of teaching for learning[52].

In the GMC model proposed here it is important to adopt assessment that supports learning towards competence and herein lies the challenge.

The term competence is widely used in vocational and pre-vocational education and Baartman et al's (2007) overview suggests that its underlying characteristic is of "connected pieces of knowledge, skills and attitudes that can be used adequately solve a problem"[53]. As outlined above, GMCs provide a model of how *mathematical* competence can be demonstrated in ways that are meaningful to students applying mathematics to solve problems in a range of vocational pathways. The model of assessment appropriate for GMCs, therefore, needs to focus on such problem solving in the context of vocational practice.

Curriculum and qualification structures for vocational education in the Netherlands have, since 2008, been competence-based and assessment has been organised around Competence Assessment Programmes. These combine traditional (e.g. timed written assessment) with new forms of assessment such as portfolio assessment. In this context Baartman et al (2007)[54] identify twelve quality criteria for competence assessment programmes that can helpfully inform assessment design for GMCs.

1. Acceptability: Are the assessment criteria, process and quality acceptable to all stakeholders?
2. Authenticity: Are assessment tasks authentic?
3. Cognitive complexity: Does the overall assessment reflect the desired level of cognitive complexity required for workplace competence?
4. Comparability: Are assessment tasks comparable and consistent?
5. Costs and efficiency: Are time and resource requirements appropriate?
6. Educational consequences: Is the assessment designed to support teaching and learning?
7. Fairness: Does the assessment allow students a variety of ways in which to demonstrate their competence?
8. Fitness for purpose: Does the assessment align well with curriculum specification, instruction and learning?
9. Fitness for self-assessment: Does the assessment support students' self-regulated learning?
10. Meaningfulness: Does the assessment have value to all stakeholders including learners, teachers, employers?
11. Reliability: Are decisions reproducible?
12. Transparency: Is the assessment clear and understandable?

These questions provide a checklist that is applicable to assessment design generally. However, the notion of authenticity applies particularly to assessment design for vocational education. In the Dutch context, where vocational education has for a long time been more closely aligned with workplaces by offering parallel experience of education and workplace activity, authentic assessment has been considered in detail. In their research into authenticity of assessment in vocational education and training (VET) Gulikers and colleagues[55] considered authentic assessment to be that which engages students in activity that resembles “students’ (future) professional practice.” The implication for the assessment of GMCs, which have been designed to capture the essence of authentic use and application of mathematics in workplace activity, is that the assessment needs to mirror this important feature of their design.

In this regard, Ashford-Rowe and colleagues[56] in their design research that investigated the development of authentic assessment used eight key questions based on their reviews of the literature:

1. To what extent does the assessment activity challenge the student?
2. Is a performance, or product, required as a final assessment outcome?
3. Does the assessment activity require that transfer of learning has occurred, by means of demonstration of skill?
4. Does the assessment activity require that metacognition is demonstrated?
5. Does the assessment require a product or performance that could be recognised as authentic by a client or stakeholder? (accuracy)
6. Is fidelity required in the assessment environment? And the assessment tools (actual or simulated)?
7. Does the assessment activity require discussion and feedback?
8. Does the assessment activity require that students collaborate?

It is the intention of the GMCs that tasks designed to support student learning will demonstrate such authenticity. The assessment should therefore reflect these key principles of the GMCs. Assessment tasks based entirely on knowledge recall, for example, will be insufficient. Rather, modes of assessment should ensure that students are engaged in more complex activity that involves problem solving in vocationally relevant contexts.

It is important to draw attention to a particular aspect of authentic, vocationally-focused activity that involves mathematics raised by question 6. The implication of the requirement for authentic activity to show fidelity to the environment and engage with authentic tools strongly suggests that appropriate technology with suitable software should be available in, and integral to, the assessment. In particular, this means that across a number of GMCs the use of spreadsheets should be expected in the assessment

It is also important that the assessment design considers issues in relation to the assessment of problem solving in mathematics. This was the subject of an ACME Report in 2016[57] that considered the desirable characteristics of the assessment of problem solving and which concluded that appropriate tasks should:

- be varied in presentation (including being based on authentic scenarios);
- require students to make choices about methods to use;
- require mathematical thinking in ways that require synthesis of mathematical ideas;
- include those that lead to a range of different possible solutions, involve the interpretation of solutions, and require engagement in communication of solutions;
- allow for critical analysis, evaluation, revision and refinement of approaches and outcomes.

This offers another checklist relating for the design of assessment items in terms of their problem solving demand.

The discussion above shows that the assessment of mathematics in T Levels should reflect the modes of students’ learning of the GMCs. Portfolio Assessment is a model of assessment that is likely to ensure such an outcome. Assessment of this form entails the development of an assessed collection of student work that demonstrates their competence through tasks they have tackled during their programme of study. We are not proposing such an approach for a number of reasons, most important of which is concerned with establishing the credibility of the assessment and public confidence in the process and outcomes. Research into the use of portfolio assessment[58] points to many benefits, particularly at the local level, where it "has been associated with greater enthusiasm for teaching, higher expectations for students, and desired changes in educational goals, content and instructional procedures"[59]. However, the issues associated with establishing public credibility, together with the challenges of implementation and developing teacher expertise in a new mode of assessment, suggests that this would be a difficult model to establish.

As with competence-based assessment of VET curricula in the Netherlands, a hybrid model of assessment would be preferable. Such a model need not be constrained by traditional modes of marking. For example, the assessment of GMC activity might draw on comparative judgement methods which have been explored in some detail in mathematics by Jones and colleagues[60].

In summary, the design and implementation of the assessment of mathematics in T Levels will require careful attention. It will need to:

- support teaching and learning that has fidelity to GMCs;
- provide authentic student experiences of mathematics at the boundaries of school and work;
- focus on what it means to be mathematically competent;
- involve students in meaningful problem-solving activity;
- have meaning and credibility with all stakeholders including learners, teachers and employers.

Accordingly, there is a need for a programme of design research, implementation and development that will run for a number of years. This needs to be strategically designed from the outset to ensure iterative cycles of improvement. It is important to recognize that when introducing an innovative solution to a longstanding problem (with a history of multiple unsuccessful solutions) the first design is seen as such that – a first attempt - with the intention that over time assessment approaches can be developed that have high level of validity, credibility and authenticity.

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