

**RS ACME Primary and early years expert panel perspective:  
Spatial reasoning  
June 2024**

## **1. Executive summary**

- **Spatial reasoning** involves understanding and visualising spatial relations and the spatial properties of objects, including spatial aspects of quantities.
- **Spatial reasoning** is wider than traditional geometry, including aspects of space and shape such as position and direction, navigation, perspective-taking, scaling, transformations, shape properties and structure, composition and decomposition.<sup>1</sup>
- **Spatial reasoning** predicts mathematics performance at any age. Research shows that teaching children to think and work spatially results in substantially improved mathematics performance, with lifelong benefits.
- **Spatial reasoning** is supported by spatial representation, including manipulating objects and mental images, spatial language, gesture, sketching, maps and graphic representations.
- **Spatialising the mathematics curriculum** by emphasising thinking and working spatially has broad benefits for mathematics, including geometry, measures, number, algebra and statistics.
- **Spatial reasoning is important** in everyday life, for interpreting data and solving problems in a range of contexts, as well as for learning mathematics. It is especially important in STEM careers, where data science is of increasing importance. However, spatial thinking is a weakness for English 15-year-olds.<sup>2</sup> Spatialising the mathematics curriculum would improve the skills of the STEM workforce and help the UK to move out of the 'slow lane'.<sup>3</sup>
- **Everyone's spatial reasoning** can be improved, in a range of ways, at any age. Spatial reasoning develops as a result of genetic and developmental factors, but also environmental factors. Effective teaching strategies use spatial language, gesture and representations, with activities involving physical movement, manipulatives, robots and IT. Puzzles and problems foster visualisation, prediction and experimentation.

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<sup>1</sup> OECD, Organisation for Economic Co-operation and Development (2022). *PISA 2022 Mathematics Framework*. PISA, OECD Publishing, Paris. <https://pisa2022-maths.oecd.org/>

<sup>2</sup> OECD, Organisation for Economic Co-operation and Development (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*, PISA, OECD Publishing, Paris. [https://www.oecd-ilibrary.org/education/pisa-2022-results-volume-i\\_53f23881-en](https://www.oecd-ilibrary.org/education/pisa-2022-results-volume-i_53f23881-en)

<sup>3</sup> Royal Society (2023). *A new approach to mathematics and data education: A discussion paper from the Mathematical Futures Board of The Royal Society's Advisory Committee on Mathematics Education (ACME)*. <https://royalsociety.org/topics-policy/projects/mathematical-futures/>

- **A spatialised mathematics curriculum** is particularly helpful for disadvantaged groups, who may lack spatial experiences or vocabulary and underperform in spatial tasks. It can engage children with mathematics in different ways, drawing on interests, aptitudes and out of school experiences, in a more accessible and relevant curriculum. It can reduce the indifference and anxiety many people feel towards mathematics, including teachers as well as pupils.

## 2. Recommendations

Teaching spatial reasoning deserves a greater focus in primary mathematics, as research suggests that it has an important role in developing mathematical thinking and understanding. We may be neglecting to teach a key way of thinking mathematically which could support children at risk of underachieving. As Verdine et al. (2017:110)<sup>4</sup> suggest, ‘optimizing spatial performance may be an underutilised route to improving mathematics achievement’. This is an opportunity which should not be missed.

If we are to improve children’s spatial reasoning, the following changes are necessary:

1. **increased time allocation** in the curriculum for teaching spatial reasoning, expanding what is currently taught as geometry, e.g. including scale, perspective and navigation.
2. **pedagogical approaches** which use practical and embodied experiences, spatial images, drawing and visualisation to teach number, measures and data handling.
3. **a broader view of mathematics**, linking to other curriculum areas, e.g. design and technology, computing, geography, science, art and physical education.

Changing the curriculum requires a policy vision of mathematics as a broader, more accessible and relevant subject. It follows that:

- **assessment** content and approaches must be changed to assess spatial reasoning.
- **guidance and resources** from official sources are needed to support spatialised pedagogy.
- **professional development**, including initial teacher education (ITE), should increase its focus on spatial reasoning.

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<sup>4</sup> Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N.S. (2017). Links between Spatial and Mathematical Skills across the Preschool Years. *Monographs of the Society for Research in Child Development*, **82**(1): 1–150.  
<https://srcd.onlinelibrary.wiley.com/toc/15405834/2017/82/1>

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## 3. Introduction to spatial reasoning

### The Program for International Student Assessment (PISA) definition of Space and Shape<sup>5</sup>

Space and Shape encompasses a wide range of phenomena that are encountered everywhere in our visual and physical world: patterns, properties of objects, positions and orientations, representations of objects, decoding and encoding of visual information, navigation and dynamic interaction with real shapes as well as with representations, movement, displacement, and the ability to anticipate actions in space.

Geometry serves as an essential foundation for space and shape, but the category extends beyond traditional geometry in content, meaning and method, drawing on elements of other mathematical areas such as spatial visualisation, measurement and algebra. For instance, shapes can change and a point can move along a locus, thus requiring function concepts.

Measurement formulae are central in this area.

The recognition, manipulation and interpretation of shapes in settings that call for tools ranging from dynamic geometry software to Global Positioning Systems (GPS), and to machine learning software are included in this content category.

### 3.1. What is spatial reasoning?

Spatial reasoning is the ability to understand and visualise spatial properties of objects and spatial relations, including spatial properties of quantities. It can be supported by spatial enactment and representation, including manipulating physical objects and mental images, spatial language, gesture, drawing, maps, diagrams and graphs. Spatial reasoning involves many different aspects, including shape properties and structure of objects, composition and decomposition of shapes, position, direction, movement and rotation, symmetry, perspective-taking, navigation and scaling.<sup>6</sup>

One example of using spatial reasoning is this task: **Can you predict which of these nets will fold up into a pyramid?**

<sup>5</sup> OECD, Organisation for Economic Co-operation and Development (2022). *PISA 2022 Mathematics Framework*. PISA, OECD Publishing, Paris. <https://pisa2022-maths.oecd.org/>

<sup>6</sup> Gifford, S., Gipton, C., Williams, H.J., Lancaster, A., Bates, K.E., Williams, A.Y., Gilligan-Lee, K., Borthwick, A., & Farran, E.K. (2022). *Spatial Reasoning in early childhood*. Early Childhood Mathematics Group. <https://doi.org/10.31234/osf.io/jnwpu>

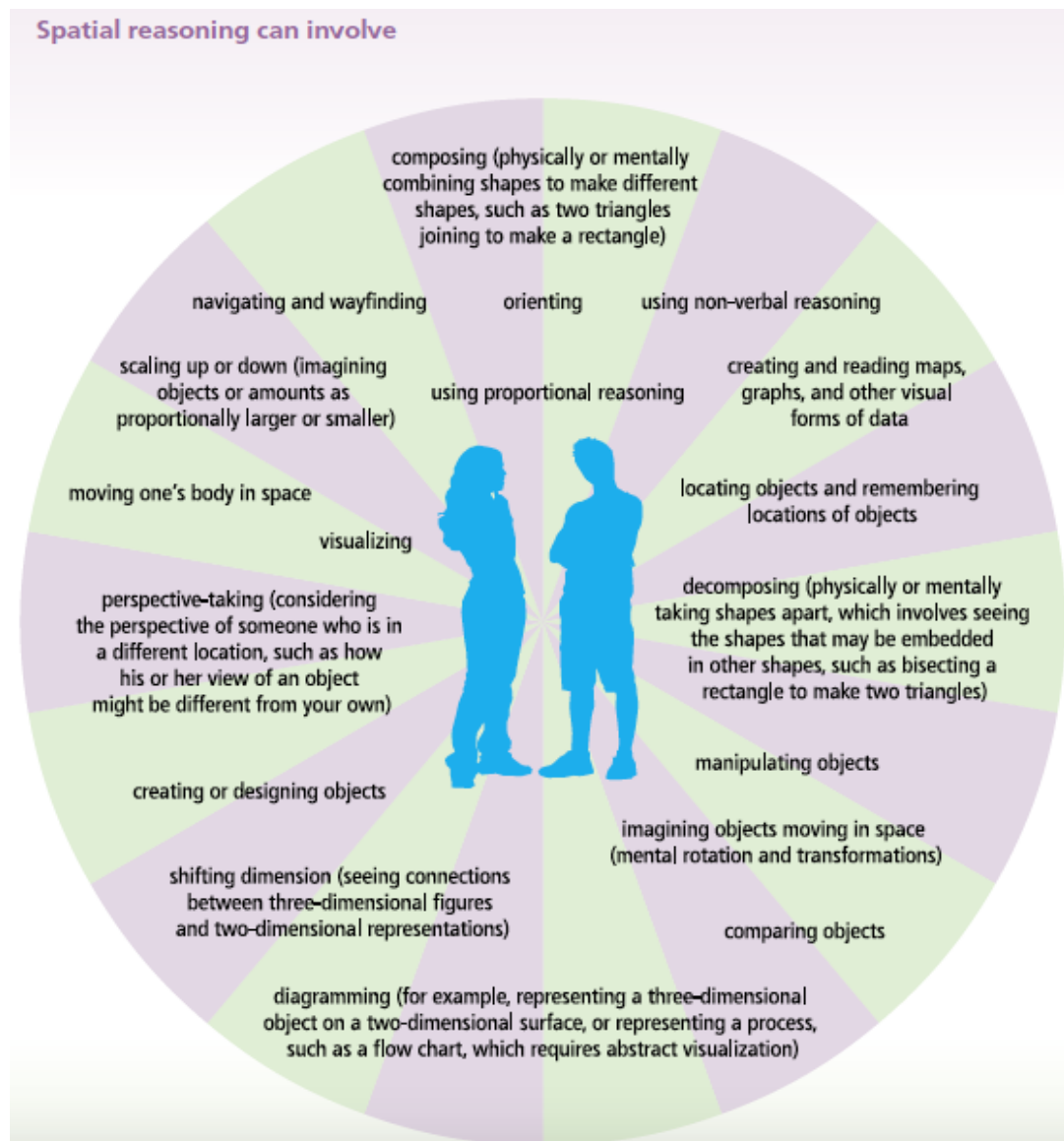


This requires mentally folding the shapes and reasoning along the lines of ‘that won’t work because those ones will overlap’, but thinking in images rather than words. The reasoning is non-verbal, requiring visualisation skills.

According to the Ontario Ministry of Education (2014)<sup>7</sup>, who have devised a curriculum from early years to Grade 12, spatial reasoning involves many kinds of thinking which are important life skills as well as mathematically. For instance, it involves mentally manipulating objects, scaling, proportional reasoning, designing, diagramming and reading maps and graphs.

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<sup>7</sup> Ontario Ministry of Education (2014). *Paying attention to spatial reasoning. K-12*. Services Ontario. <http://www.edu.gov.on.ca/eng/literacynumeracy/Inspayingattention.pdf>



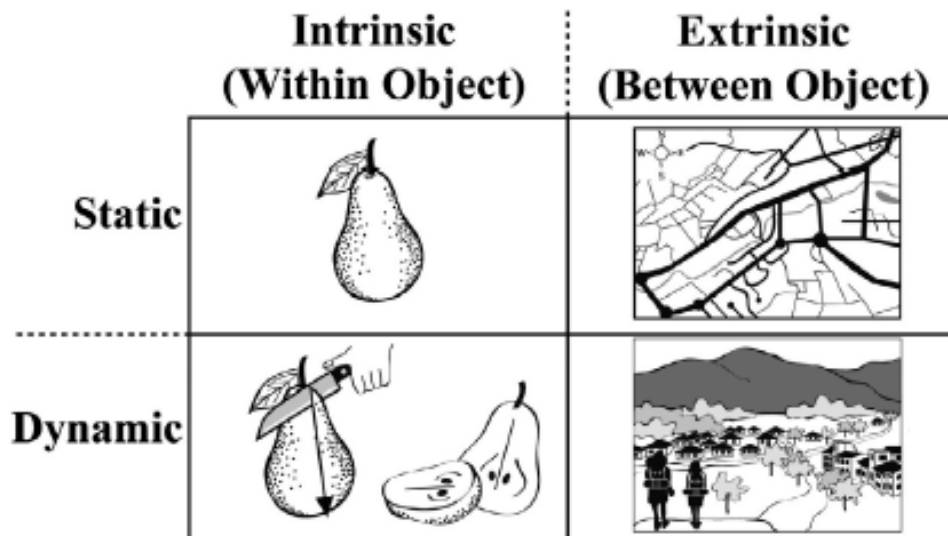
Ontario Ministry of Education (2014) *Paying attention to spatial reasoning, K-12*, Services Ontario<sup>8</sup>

Traditionally, early geometry has distinguished between 'space and 'shape', with the former focusing on 'position and direction'.<sup>9</sup> The terms 'space and shape' are also used by PISA.<sup>10</sup> Psychological studies tend to make similar distinctions between 'intrinsic' and 'extrinsic' aspects of spatial thinking respectively, both of which may be static or dynamic, as shown below.

<sup>8</sup> Ontario Ministry of Education (2014). *Paying attention to spatial reasoning. K-12*. Services Ontario. <http://www.edu.gov.on.ca/eng/literacynumeracy/Inspayingattention.pdf>

<sup>9</sup> DfE, Department for Education. (2013). *Mathematics Programmes of Study: National Curriculum in England*. <https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study>

<sup>10</sup> OECD, Organisation for Economic Co-operation and Development (2022). *PISA 2022 Mathematics Framework*. PISA, OECD Publishing, Paris. <https://pisa2022-maths.oecd.org/>



**Figure 1. Proposed typology of spatial skills.**

Figure re-used with permission from Newcombe (2018)<sup>11</sup>

Newcombe (2018)<sup>11</sup> argues that the intrinsic/extrinsic distinction is useful because these skills involve different parts of the brain. For instance, ‘intrinsic’ skills include rotating shapes, whereas ‘extrinsic’ skills involve navigation: these are different kinds of cognitive activities.

### 3.2. Why is spatial reasoning important?

Spatial reasoning is involved in many everyday tasks, such as storage problems, route finding and following assembly diagrams.<sup>12</sup> It is also strongly related to STEM careers as shown in the graph below: those with the highest spatial scores in US secondary school were later employed in engineering, mathematics and computer science.<sup>13</sup>

<sup>11</sup> Newcombe, N.S. (2018). Three kinds of spatial cognition. *Stevens' Handbook of Experimental Psychology and Cognitive Neuroscience*, 3: 1–31. <https://doi.org/10.1002/9781119170174.epcn315>

<sup>12</sup> Farran, E.K. (2019). Spatial ability as a gateway to STEM success. *Impact*, 6, [https://my.chartered.college/impact\\_article/spatial-ability-as-a-gateway-to-stem-success/](https://my.chartered.college/impact_article/spatial-ability-as-a-gateway-to-stem-success/)

<sup>13</sup> Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N.S. (2017). Links between Spatial and Mathematical Skills across the Preschool Years. *Monographs of the Society for Research in Child Development*, 82(1): 1–150. <https://srcd.onlinelibrary.wiley.com/toc/15405834/2017/82/1>

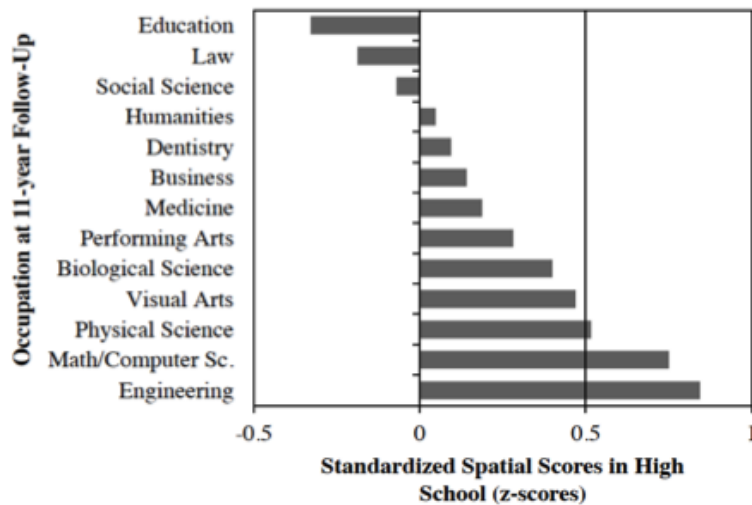


Figure 2. Spatial scores in 9<sup>th</sup>-12<sup>th</sup> grade and reported occupations 11 years later (derived from Wai et al., 2009).

Figure re-used with permission from Verdine et al. (2017)<sup>13</sup>

Uttal and Cohen (2012)<sup>14</sup> suggest that students ‘who cannot think well spatially’ tend to drop out of STEM college courses and that early training would prevent this. Lowrie et al. (2020)<sup>15</sup> argue that there has been increased attention to spatial reasoning recently because it is essential for the STEM workforce and ‘the complex spatial problem solving...in today’s dynamic and digital environments’. The current employment revolution has also seen an increase in the use of data across many careers and thus data science is increasing in importance: much of this is underpinned by spatial reasoning. This suggests that the introduction of spatial reasoning into the mathematics curriculum would improve the skills of both the STEM and non-STEM workforce and help the UK to move out of the ‘slow lane’.<sup>16</sup>

Spatial thinking is strongly correlated with high mathematics performance, according to Mix & Cheng (2012)<sup>17</sup>: “The relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related”.

<sup>14</sup> Uttal, D. H. & Cohen, C. A. (2012). Spatial thinking and STEM education: When, why, and how? In B. H. Ross (Ed.), *Psychology of Learning and Motivation* (pp. 147–181). Elsevier Academic Press. <https://doi.org/10.1016/B978-0-12-394293-7.00004-2>

<sup>15</sup> Lowrie, T., Resnick, I., Harris, D., & Logan, T. (2020). In search of the mechanisms that enable transfer from spatial reasoning to mathematics understanding. *Mathematics Education Research Journal*, **32**: 175–188. <https://doi.org/10.1007/s13394-020-00336-9>

<sup>16</sup> Royal Society (2023). *A new approach to Mathematics and data education: A discussion paper from the Mathematical Futures Board of The Royal Society’s Advisory Committee on Mathematics Education (ACME)*. <https://royalsociety.org/topics-policy/projects/mathematical-futures/>

<sup>17</sup> Mix, K.S. & Cheng, Y.L. (2012) The relation between space and math: developmental and educational implications. *Advances in Child Development and Behavior*, **42**: 197–243. <https://doi.org/10.1016/B978-0-12-394388-0.00006-X>



Spatial ability is predictive of mathematics ability in the primary and early years.<sup>18–22</sup> There is a substantial overlap between spatial and mathematical reasoning; besides geometry, many aspects of mathematics include spatial aspects, such as measurement, place value, fractions, proportional reasoning, algebra and statistics. Spatial models and images are used to represent number relationships, e.g. number lines, arrays and bar models. Spatial reasoning is therefore required for much mathematical understanding; those children who do not easily comprehend visual images, or fail to draw on visualisation as a problem solving strategy, may be at a disadvantage.

## 4. Improving spatial reasoning

### 4.1. Everyone's spatial thinking can be improved

Spatial reasoning can be taught, at any age, to both sexes and using a range of approaches.<sup>23</sup> Some people may be better spatial thinkers than others, but all can improve, and spatial training seems particularly effective for children from disadvantaged backgrounds, who may lack early spatial experiences.<sup>24,25</sup> If spatial reasoning skills affect mathematics, then teaching these is an equity issue. Girls and children from 'low-income homes'<sup>26,27</sup> are 'harmed in their progression in mathematics' by having fewer opportunities to develop spatial reasoning.<sup>28</sup> For some children, spatial approaches may provide an additional, non-verbal route to mathematics.

### 4.2. How does spatial reasoning develop?

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<sup>18</sup> Gilligan, K.A., Flouri, E., & Farran, E.K. (2017). The contribution of spatial ability to mathematics achievement in middle childhood. *Journal of Experimental Child Psychology*, **163**: 107–125. <https://doi.org/10.1016/j.jecp.2017.04.016>

<sup>19</sup> Gilligan, K.A., Hodgkiss, A., Thomas, M.S., & Farran, E.K. (2019). The developmental relations between spatial cognition and mathematics in primary school children. *Developmental Science*. **22**(4): e12786. <https://doi.org/10.1111/desc.12786>

<sup>20</sup> McDougal, E., Silverstein, P., Treleaven, O., Jerrom, L., Gilligan-Lee, K.A., Gilmore, C., & Farran, E.K. (2023). Associations and indirect effects between LEGO® construction and mathematics performance. *Child Development*, **94**(5): 1381–1397. <https://doi.org/10.1111/cdev.13933>

<sup>21</sup> Mix, K.S. (2019). Why are spatial skills and mathematics related? *Child Development Perspectives*, **13**(2): 121–126. <https://doi.org/10.1111/cdep.12323>

<sup>22</sup> Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N.S. (2017). Links between Spatial and Mathematical Skills across the Preschool Years. *Monographs of the Society for Research in Child Development*, **82**(1): 1–150. <https://srcd.onlinelibrary.wiley.com/toc/15405834/2017/82/1>

<sup>23</sup> Uttal, D.H., Meadow, N.G., Newcombe, N.S., Tipton, E., Hand, L.L., Alden, A.R., & Warren, C. (2013). The malleability of spatial skills: a meta-analysis of training studies. *Psychological Bulletin*, **139**(2): 352–402. <https://doi.org/10.1037/a0028446>

<sup>24</sup> Bower, C., Zimmermann, L., Verdine, B., Toub, T.S., Islam, S., Foster, L., Evans, N., Odean, R., Cibischino, A., Pritulsky, C., Hirsh-Pasek, K., & Golinkoff, R.M. (2020). Piecing together the role of a spatial assembly intervention in preschoolers' spatial and mathematics learning: Influences of gesture, spatial language, and socioeconomic status. *Developmental Psychology*, **56**(4): 686–698. ISSN: 0012-1649. <https://doi.org/10.1037/dev0000899>

<sup>25</sup> Schmitt, S.A., Korucu, I., Napoli, A.R., Bryant, L.M., & Purpura, D.J. (2018). Using block play to enhance preschool children's mathematics and executive functioning: A randomized controlled trial. *Early Childhood Research Quarterly*, **44**: 181–191. <https://doi.org/10.1016/j.ecresq.2018.04.006>

<sup>26</sup> Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N.S. (2017). Links between Spatial and Mathematical Skills across the Preschool Years. *Monographs of the Society for Research in Child Development*, **82**(1): 1–150. <https://srcd.onlinelibrary.wiley.com/toc/15405834/2017/82/1>

<sup>27</sup> Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N.S. (2014). Finding the missing piece: Blocks, puzzles, and shapes fuel school readiness. *Trends in Neuroscience & Education*, **3**(1), 7–13. <https://doi.org/10.1016/j.tine.2014.02.005>

<sup>28</sup> Sarama, J. & Clements, D.H. (2009). *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children*. NY & London: Routledge. <https://doi.org/10.4324/9780203883785>



Spatial reasoning depends on awareness of spatial properties and spatial relations, which develop from sensorimotor experiences. It involves the representation of these properties and relations mentally, verbally and graphically. Babies begin by developing awareness of their bodies in movement, and of shapes and distances. Large-scale movement and toys help toddlers to develop spatial memory, language and understanding of large-scale space and perspective.<sup>29</sup>

Identifying, visualising and predicting transformations develop gradually in the early years.<sup>30</sup> Very young children can manipulate mental images by sliding, four-year-olds can begin to imagine turning and flipping shapes and six-year-olds can recognise mirror-image reversals.

Young children can understand scale models, as in small world play, for instance with toy farms or train sets. They gradually understand and make 2D representations of 3D shapes and spaces, for instance by drawing constructions and pictorial maps (e.g. <sup>31</sup>). Older children can construct models shown from different viewpoints<sup>32</sup> and use maps in more sophisticated ways, such as orienteering.<sup>33</sup> Throughout the primary school years, children develop more complex spatial skills and understanding involving mental manipulations, symmetry, perspective, scaling and proportion.<sup>34</sup>

Some aspects of spatial reasoning, which require processing of complex information, are dependent on development. For instance, orienteering requires constant reinterpretation of a map in relation to a person's changing position and view of the surroundings, which is too cognitively demanding for young children. However, the amount and range of physical experiences and adult support are highly influential, so individual children's spatial capabilities will vary. Parents' use of precise spatial language with very young children has a long-term effect on spatial skills. Pruden et al. (2011)<sup>35</sup> suggest that language enables children to encode spatial properties and relations, thereby freeing working memory. Many studies have found that gesturing by both adults and children helps spatial learning, particularly for children from disadvantaged backgrounds (e.g. <sup>36</sup>).

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<sup>29</sup> Oudgenoeg-Paz, O., Leseman, P.P.M., & Volman, M.(C).J.M. (2015). Exploration as a mediator of the relation between the attainment of motor milestones and the development of spatial cognition and spatial language. *Developmental Psychology*, **51**(9): 1241–1253. <https://doi.org/10.1037/a0039572>

<sup>30</sup> Clements, D.H. & Sarama, J. (2021). *Learning and Teaching Early Math: The Learning Trajectories Approach*. NY & London: Routledge

<sup>31</sup> Frick, A. & Newcombe, N. (2012). Getting the big picture: Development of spatial scaling abilities. *Cognitive Development*, **27**(3): 270–282. <https://doi.org/10.1016/j.cogdev.2012.05.004>

<sup>32</sup> McDougal, E., Silverstein, P., Treleaven, O., Jerrom, L., Gilligan-Lee, K.A., Gilmore, C., & Farran, E.K. (2023). Associations and indirect effects between LEGO® construction and mathematics performance. *Child Development*, **94**(5): 1381–1397. <https://doi.org/10.1111/cdev.13933>

<sup>33</sup> Sarama, J. & Clements, D.H. (2009). *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children*. NY & London: Routledge. <https://doi.org/10.4324/9780203883785>

<sup>34</sup> Hodgkiss, A., Gilligan-Lee, K.A., Thomas, M.S.C., Tolmie, A.K., & Farran, E.K. (2021). The developmental trajectories of spatial skills in middle childhood. *British Journal of Developmental Psychology*, **39**(4): 566–683. <https://doi.org/10.1111/bjdp.12380>

<sup>35</sup> Pruden, S.M., Levine, S.C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter? *Developmental Science*, **14**(6): 1417–1430. <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3372906/>

<sup>36</sup> Bower, C., Zimmermann, L., Verdine, B., Toub, T.S., Islam, S., Foster, L., Evans, N., Odean, R., Cibischino, A., Pritulsky, C., Hirsh-Pasek, K., & Golinkoff, R.M. (2020). Piecing together the role of a spatial assembly intervention in preschoolers' spatial

Therefore, the development of children’s spatial skills will depend on age, but may also vary considerably with experience.

### 4.3. How spatial reasoning helps mathematical learning

There are very strong correlations between spatial reasoning and mathematics attainment; for instance, there are correlations between later mathematics and early puzzle play and construction. Furthermore, spatial training studies robustly demonstrate that this relationship is causal.<sup>37</sup> This may be because both spatial and numerical thinking activate the same brain area.<sup>38</sup> In addition, visualising spatial models of number relationships helps understanding. According to Sinclair and Bruce (2015:321)<sup>39</sup> ‘...Geometry provides mathematics with its basic meanings (through representations, models, visualizations, analogies and physical materials)’. One important spatial image of number relations is the number line, which can be extended to show large or negative numbers or expanded to show fractions.<sup>40</sup> Spatial training has been found to have a major impact on children’s development of a mental number line,<sup>41</sup> showing that improving spatial thinking helps children to think spatially about number relationships.

There is agreement that visualisation is a key element of spatial reasoning. Studies of spatial skills usually involve tasks like visualising rotation and other transformations, including combining, cutting and folding shapes (e.g. <sup>42</sup>). Gilligan et al. (2019)<sup>43</sup> also found that spatial scaling and disembedding skills correlated with the mathematics performance of primary school children. Giles et al. (2018)<sup>44</sup> claimed that ball skills predicted children’s mathematics, finding that ‘mathematical attainment in children is related to interceptive-timing ability’. This suggests that judging speed as well as predicting trajectories of body parts and moving objects may be part of spatial reasoning and also connected to mathematical thinking. It seems that spatial reasoning may include a range of skills linked to mathematics, some of which are yet to be investigated by research.

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and mathematics learning: Influences of gesture, spatial language, and socioeconomic status. *Developmental Psychology*, **56**(4): 686–698. ISSN: 0012-1649. <https://doi.org/10.1037/dev0000899>

<sup>37</sup> Hawes, Z.C.K., Gilligan-Lee, K.A., & Mix, K.S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology*, **58**(1): 112–137. <https://doi.org/10.1037/dev0001281>

<sup>38</sup> Hawes, Z., Sokolowski, H.M., Ononye, C.B., & Ansari, D. (2019) Neural underpinnings of numerical and spatial cognition: An fMRI meta-analysis of brain regions associated with symbolic number, arithmetic, and mental rotation. *Neuroscience & Biobehavioural Reviews*, **103**: 316–336. <https://doi.org/10.1016/j.neubiorev.2019.05.007>

<sup>39</sup> Sinclair, N. & Bruce, C.D. (2015). New opportunities in geometry education at the primary school. *ZDM Mathematics Education*, **47**: 319–329. <https://doi.org/10.1007/s11858-015-0693-4>

<sup>40</sup> Hawes, Z.C.K., Gilligan-Lee, K.A., & Mix, K.S. (2023). Infusing spatial thinking into elementary and middle school mathematics: What, why, and how? In: Robinson, K.M., Dubé, A.K., & Kotsopoulos, D. (Eds.) *Mathematical Cognition and Understanding*. Cham: Springer. [https://doi.org/10.1007/978-3-031-29195-1\\_2](https://doi.org/10.1007/978-3-031-29195-1_2)

<sup>41</sup> EEF, Education Endowment Foundation (2017). *Improving Mathematics in Key Stages 2 and 3*. <https://educationendowmentfoundation.org.uk/education-evidence/guidance-reports/maths-ks-2-3>

<sup>42</sup> Hawes, Z. & Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychonomic Bulletin & Review*. **27**:465–482. <https://doi.org/10.3758/s13423-019-01694-7>

<sup>43</sup> Gilligan, K.A., Hodgkiss, A., Thomas, M.S., & Farran, E.K. (2019). The developmental relations between spatial cognition and mathematics in primary school children. *Developmental Science*. **22**(4): e12786. <https://doi.org/10.1111/desc.12786>

<sup>44</sup> Giles, O.T., Shire, K.A., Hill, J.B., Mushtaq, F., Waterman, A., Holt, R.J., Culmer, P.R., Williams, J.H.G., Wilkie, R.M., & Mon-Williams, M. (2018). Hitting the target: mathematical attainment in children is related to interceptive-timing ability. *Psychological Science*. **29**(8): 1334–1345. <https://doi.org/10.1177/0956797618772502>

Spatial reasoning also facilitates the learning of new concepts: people intuitively use spatial analogies or representations, including gesture, before adopting more abstract, verbal or symbolic representations.<sup>45</sup> This relates to theories of the embodied mind and learning, which point out that many mathematics concepts are grounded in physical interaction with the world.<sup>46</sup> For example, we refer to high and low numbers, and horizontal lines. Gerofsky (2011)<sup>47</sup> found that secondary students who could act out ‘being in a graph’, with the centre of their body as the origin, had a deeper understanding of a graph than those who just visualised it. Sung et al. (2017)<sup>48</sup> found that young children who moved along a floor number line had improved number line understanding, which also enabled them to code computer games. This suggests that enacting spatial representations will help children understand mathematical relationships, as well as using gesture and manipulatives. It fits with ideas from Bruner (1966)<sup>49</sup> about the role of concrete and visualised experience in developing abstract understanding (see also <sup>50</sup>). This is supported by evidence from neuroscience, that using multiple, kinaesthetic and multisensory modes of representation enables ideas to be memorised through networked associations, building deeper understanding.<sup>51,52</sup> Therefore, a more spatial, embodied approach, including teaching children visualising skills based on practical experience, might help in the initial stages of teaching any aspect of mathematics.

## 5. Teaching spatial reasoning

### 5.1. How should we teach spatial reasoning?

A range of activities have been suggested to teach spatial reasoning. Recent evidence suggests that practical activities are more effective than computerised or paper-based training, although these may have their place.<sup>53</sup> Effective activities include shape puzzles and construction activities: these focus attention on shape properties as well as transformations

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<sup>45</sup> Mix, K.S. (2019). Why are spatial skills and mathematics related? *Child Development Perspectives*, **13**(2): 121–126. <https://doi.org/10.1111/cdep.12323>

<sup>46</sup> Lakoff, G. & Núñez, R.E. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. Basic Books.

<sup>47</sup> Gerofsky, S. (2011) Seeing the graph vs. being the graph. In: Stam, G. & Ishino, M. (Eds.) *Integrating Gestures: The Interdisciplinary Nature of Gesture*. Amsterdam: John Benjamins Publishing Company. <https://doi.org/10.1075/gs.4.22ger>

<sup>48</sup> Sung, W., Ahn, J., & Black, J.B. (2017) Introducing computational thinking to young learners: Practicing computational perspectives through embodiment in mathematics education. *Technology, Knowledge and Learning*, **22**(3): 443–463. <https://doi.org/10.1007/s10758-017-9328-x>

<sup>49</sup> Bruner, J. (1966). *Toward a Theory of Instruction*. Cambridge, MA: Harvard University Press

<sup>50</sup> Griffiths, R., Gifford, S., & Back, J. (2017). *Using Manipulatives in the Foundations of Arithmetic: Literature Review*. Nuffield Foundation. <https://www.nuffieldfoundation.org/project/using-manipulatives-in-the-foundations-of-arithmetic-2>

<sup>51</sup> Goswami, U. & Bryant, P. (2007). *Children’s Cognitive Development and Learning (Primary Review Research Survey 2/1a)*, Cambridge: University of Cambridge Faculty of Education. <https://cprtrust.org.uk/wp-content/uploads/2014/06/research-survey-2-1a.pdf>

<sup>52</sup> Rogers, C. & Thomas, M.S.C. (2023). *Educational Neuroscience*. London: Routledge. <https://doi.org/10.4324/9781003185642>

<sup>53</sup> Hawes, Z.C.K., Gilligan-Lee, K.A., & Mix, K.S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology*, **58**(1): 112–137. <https://doi.org/10.1037/dev0001281>

like flipping and turning.<sup>54,55</sup> The effectiveness of spatial language and gesture is dependent on the quality of provision: for instance, more varied and unusual shapes focus attention on more specific shape properties.<sup>56,57</sup> Young children benefit from being explicitly taught strategies for solving puzzles, including visualising; this particularly helps children from disadvantaged backgrounds.<sup>58</sup> Verdine et al. (2017)<sup>59</sup> emphasise that children need to be taught ‘how to think spatially’, rather than being taught content, and recommend ‘especially goal-oriented tasks done in conjunction with knowledgeable adults’. In the early years, they recommend that guided play, rather than free play or direct instruction, elicits more spatial language, problem solving and learning (see also <sup>60</sup>). Activities might include model-making and spatial games (e.g. <sup>61</sup>).

With primary school children, a variety of modes seem effective, including outdoor activities and mapping, folding, cutting and construction, paper and pencil tasks including drawing, and directing robots (e.g. <sup>62,63</sup>). With older primary and secondary school learners, Fujita et al. (2020)<sup>64</sup> suggest that problem solving with 2D representations of 3D requires two aspects of spatial reasoning:

- **visualisation:** mental manipulations of images of shapes including rotation, transforming diagrams to another form, reorienting, drawing nets and adding additional lines;

<sup>54</sup> Levine, S.C., Ratliff, K.R., Huttenlocher, J., & Cannon, J. (2012). Early puzzle play: A predictor of preschoolers’ spatial transformation skill. *Developmental Psychology*, **48**(2): 530–542. <https://psycnet.apa.org/doi/10.1037/a0025913>

<sup>55</sup> Wolfgang, C., Stannard, L., & Jones, I. (2003). Advanced constructional play with LEGOs among preschoolers as a predictor of later school achievement in mathematics. *Early Child Development and Care*, **173**(5): 467–475. <https://doi.org/10.1080/0300443032000088212>

<sup>56</sup> Ferrara, K., Hirsh-Pasek, K., Newcombe, N.S., Golinkoff, R.M., & Lam, W.S. (2011). Block talk: Spatial language during block play. *Mind, Brain and Education*, **5**(3): 143–151. <https://doi.org/10.1111/j.1751-228X.2011.01122.x>

<sup>57</sup> Verdine, B.N., Zimmermann, L., Foster, L., Marzouk, M.A., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. (2018). Effects of geometric toy design on parent-child interactions and spatial language. *Early Childhood Research Quarterly*, **46**: 126–141. <https://doi.org/10.1016/j.ecresq.2018.03.015>

<sup>58</sup> Ribeiro, L.A., Casey, B., Dearing, E., Berg Nordahl, K., Aguiar, C., & Zachrisson, H. (2020). Early maternal spatial support for toddlers and math skills in second grade. *Journal of Cognition and Development*, **21**(2): 282–311. <https://doi.org/10.1080/15248372.2020.1717494>

<sup>59</sup> Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N.S. (2017). Links between Spatial and Mathematical Skills across the Preschool Years. *Monographs of the Society for Research in Child Development*, **82**(1): 1–150. <https://srcd.onlinelibrary.wiley.com/toc/15405834/2017/82/1>

<sup>60</sup> Skene, K., O’Farrelly, C.M., Byrne, E.M., Kirby, N., Stevens, E.C., & Ramchandani, P.G. (2022). Can guidance during play enhance children’s learning and development in educational contexts? A systematic review and meta-analysis. *Child Development*, **98**(4): 1162–1180. <https://doi.org/10.1111/cdev.13730>

<sup>61</sup> Pritulsky, C., Morano, C., Odean, R., Bower, C., Hirsh-Pasek, K., & Michnick Golinkoff, R. (2020). Spatial thinking: Why it belongs in the preschool classroom. *Translational Issues in Psychological Science*, **6**(3): 271–282. <https://doi.org/10.1037/tps0000254>

<sup>62</sup> Mulligan, J., Woolcott, G., Mitchelmore, M., Busatto, S., Jennifer, L., & Davis, B. (2020). Evaluating the impact of a Spatial Reasoning Mathematics Program (SRMP) intervention in the primary school. *Mathematics Education Research Journal*, **32**: 285–305. <https://doi.org/10.1007/s13394-020-00324-z>

<sup>63</sup> Lowrie, T., Logan, T., Harris, D., & Hegarty, M. (2018). The impact of an intervention program on students’ spatial reasoning: student engagement through mathematics-enhanced learning activities. *Cognitive Research: Principles and Implications*, **3**: 50. <https://doi.org/10.1186/s41235-018-0147-y>

<sup>64</sup> Fujita, T., Kondo, Y., Kumakura, H., Kunimune, S., & Jones, K. (2020). Spatial reasoning skills about 2D representations of 3D geometrical shapes in grades 4 to 9. *Mathematics Education Research Journal*, **32**: 235–255. <https://doi.org/10.1007/s13394-020-00335-w>

- **property-based analytic reasoning:** interpreting the structural elements of shapes and decomposing objects into their parts, using reasoning and decision-making based on knowledge of geometric properties.

They argue that children need some knowledge of geometric principles, in order, for instance, to disregard misleading distortions of shapes drawn in perspective.

There are obvious opportunities for linking spatial reasoning with other subjects, such as physical and environmental education, art and design, as well as science. These provide meaningful contexts which show applications of mathematics, as well as being likely to engage a wide range of children in further experience.

### 5.2. Current primary mathematics education neglects spatial thinking

In England, geometry is currently underemphasized in primary mathematics: the national curriculum devotes four times as many pages to number.<sup>65</sup> The geometry curriculum is divided into ‘Properties of shapes’ and ‘Position and direction’, corresponding with traditional aspects of ‘shape’ and ‘space’. This curriculum is narrow, tending to focus on classification, angles and co-ordinates, rather than promoting visualisation or cross-curricular contexts. The Year 6 ‘shape’ curriculum (for 10/11 year olds) potentially includes some visualising in drawing shapes and recognising nets, while the ‘space’ element consists of positioning and reflecting shapes on co-ordinate grids. Instead of emphasising spatial thinking, the supplementary guidance refers to algebraic expressions, e.g.  $d = 2 \times r$ ;  $a = 180 - (b + c)$ .

#### Geometry – properties of shapes

##### Statutory requirements

Pupils should be taught to:

- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

<sup>65</sup> DfE, Department for Education. (2013). *Mathematics Programmes of Study: National Curriculum in England*. <https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study>

## Geometry – position and direction

### Statutory requirements

Pupils should be taught to:

- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

Year 6 statutory curriculum (DfE, 2013)<sup>66</sup>

Overall, there is a lack of coherence and progression within Geometry: for instance, ‘Position and direction’ is missing in Year 3 (for 7-8 year olds) and is focused solely on co-ordinate grids for the remaining three years.

In practice, little time is allotted to geometry in primary schools, reflecting its marginal role in high stakes national tests. One observer noted that many schools ‘already miss out units of geometry when no-one’s looking because they know it will only be worth a few marks on a test paper’. In recent [KS2 SATs](#) about 10% of questions were on geometry, with only two requiring visualisation. Ofsted (2023)<sup>67</sup> reported that geometry was often only taught at the end of the year, after the tests.

With regard to professional development, current materials for primary teachers tend to focus on number e.g. [NCETM](#). For teacher training, our 2019 informal survey of PGCE primary programmes found on average that 2.5 out of 24 hours for mathematics were on geometry.

In the early years, there is a rather contradictory situation: in 2021, the Early Years Foundation Stage (DfE, 2021)<sup>68</sup> introduced spatial reasoning into the statutory mathematics educational programme for children from birth to five. However, *Shape, Space & Measures* were removed from the Early Learning Goals (statutory assessments for five-year-olds) reducing the incentive for reception teachers to focus on spatial learning.<sup>69</sup> Despite the statutory curriculum, Bates et al. (2022)<sup>70</sup> found that early years practitioners did not prioritise spatial reasoning in their settings.

This neglect of geometry and spatial reasoning is partly due to a national focus on number

<sup>66</sup> DfE, Department for Education. (2013). *Mathematics Programmes of Study: National Curriculum in England*.

<https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study>

<sup>67</sup> Ofsted (2023). *Coordinating Mathematical Success: The Mathematics Subject Report*.

<https://www.gov.uk/government/publications/subject-report-series-maths/coordinating-mathematical-success-the-mathematics-subject-report>

<sup>68</sup> DfE, Department for Education (2021). *Statutory Framework for the Early Years Foundation Stage*

<https://www.gov.uk/government/publications/early-years-foundation-stage-framework--2>

<sup>69</sup> EEF, Education Endowment Foundation (2019). *Early Years Foundation Stage Profile (EYFSP) Reforms: Pilot Report*.

<https://educationendowmentfoundation.org.uk/projects-and-evaluation/projects/early-years-foundation-stage-profile-pilot>

<sup>70</sup> Bates, K.E., Williams, A.E., Gilligan-Lee, K.A., Gripton, C., Lancaster, A., Borthwick, A., Gifford, S., & Farran, E.K. (2022).

Practitioner’s perspectives on spatial reasoning in educational practice from birth to 7 years. *British Journal of Educational Psychology*, 93(2): 571–590. <https://doi.org/10.1111/bjep.12579>



intervention programmes since the National Numeracy Strategy in 1999. However, these, and professional development materials, have emphasised spatial models and images e.g. NCETM (2006). Consequently, Ofsted (2023)<sup>71</sup> found that ‘Often, teachers use physical resources and pictorial representations to help pupils see underlying mathematical structures’. Therefore, spatial thinking and representations have been recommended as a way of visualising number relationships.

According to PISA results, spatial thinking is a weakness for UK 15-year-olds; high performing countries have relative strengths in shape and space.<sup>72</sup> Sorby & Panther (2020)<sup>73</sup> argue that improving spatial skills would therefore mean that ‘citizens are better-prepared for everyday life in our rapidly changing technological society’. Clements & Sarama (2021)<sup>74</sup> report that US pre-school children perform less well on visualisation and imagery tasks than children in countries such as Japan and China, suggesting those countries use more visual representation and expect more drawing from an early age. Recently, there has been a call for more emphasis on spatial reasoning internationally,<sup>75</sup> with corresponding curricular developments in Ontario and Australia (e.g. <sup>76,77</sup>).

### 5.3. The future spatial mathematics curriculum

A curriculum to develop spatial reasoning needs to include broader aspects than in traditional geometry, such as perspective-taking, scaling and navigation; it would also develop visualising skills, such as mental rotation, which support understanding and prediction. Cultural applications of geometry within other subjects, such as geography, science, art and design technology, would provide contexts likely to engage a range of children.

The current primary national curriculum is not merely in need of review because it is ten years old: the [2009 ‘Rose’ curriculum](#) proposals for Geometry seem more up to date, although written 15 years ago. The Rose curriculum, which was widely researched but rejected by a new government, included ICT, maps and models, as well as visualising, practical problem solving and cultural contexts. This seems more likely to engage primary children from diverse

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<sup>71</sup> Ofsted (2023). *Coordinating Mathematical Success: The Mathematics Subject Report*.

<https://www.gov.uk/government/publications/subject-report-series-maths/coordinating-mathematical-success-the-mathematics-subject-report>

<sup>72</sup> OECD, Organisation for Economic Co-operation and Development (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*, PISA, OECD Publishing, Paris. [https://www.oecd-ilibrary.org/education/pisa-2022-results-volume-i\\_53f23881-en](https://www.oecd-ilibrary.org/education/pisa-2022-results-volume-i_53f23881-en)

<sup>73</sup> Sorby, S.A. & Panther, G.C. (2020). Is the key to better PISA math scores improving spatial skills? *Mathematics Education Research Journal*, **32**: 213–233. <https://doi.org/10.1007/s13394-020-00328-9>

<sup>74</sup> Clements, D.H. & Sarama, J. (2021). *Learning and Teaching Early Math: The Learning Trajectories Approach*. NY & London: Routledge

<sup>75</sup> Sinclair, N. & Bruce, C.D. (2015). New opportunities in geometry education at the primary school. *ZDM Mathematics Education*, **47**: 319–329. <https://doi.org/10.1007/s11858-015-0693-4>

<sup>76</sup> Mulligan, J., Woolcott, G., Mitchelmore, M., Busatto, S., Jennifer, L., & Davis, B. (2020). Evaluating the impact of a Spatial Reasoning Mathematics Program (SRMP) intervention in the primary school. *Mathematics Education Research Journal*, **32**: 285–305. <https://doi.org/10.1007/s13394-020-00324-z>

<sup>77</sup> Lowrie, T., Logan, T., Harris, D., & Hegarty, M. (2018). The impact of an intervention program on students’ spatial reasoning: student engagement through mathematics-enhanced learning activities. *Cognitive Research: Principles and Implications*, **3**: 50. <https://doi.org/10.1186/s41235-018-0147-y>



backgrounds and to help them to learn through physical experience and spatial thinking.

EARLY	MIDDLE	LATER
<b>GEOMETRY</b>		
<p><b>E12.</b> to identify, group, match, sort and compare common shapes<sup>22</sup> using geometric properties<sup>23</sup></p> <p><b>E13.</b> to identify, reproduce and generate geometric patterns including the use of practical resources and ICT</p> <p><b>E14.</b> to generate instructions for straight and turning movement<sup>24</sup></p>	<p><b>M20.</b> to recognise symmetry properties of 2D shapes and patterns</p> <p><b>M21.</b> to make simple scalings<sup>25</sup> of objects and drawings</p> <p><b>M22.</b> to understand and use angle as the measure of turn</p> <p><b>M23.</b> to understand perimeter as a length and to find the perimeter of rectangles and other shapes</p> <p><b>M24.</b> to create sequences of instructions using ICT, including generating symmetric and repeating geometric patterns</p>	<p><b>L21.</b> to use and make maps, scale models and diagrams for a purpose</p> <p><b>L22.</b> to understand area as the space enclosed by a perimeter on a plane, and find areas of rectangles and related shapes<sup>26</sup></p> <p><b>L23.</b> to solve practical problems involving 3D objects<sup>27</sup></p> <p><b>L24.</b> to visualise geometric objects<sup>28</sup> and to recognise and make 2D representations of 3D shapes</p> <p><b>L25.</b> to create and refine sequences of instructions, using ICT to construct and explore geometric patterns and problems<sup>29</sup></p> <p><b>L26.</b> to explore aspects of geometry to find out about its origins<sup>30</sup>, and its use in different cultures, religions, art and architecture<sup>31</sup></p>

'Rose' primary curriculum for Geometry, 2009<sup>78</sup>

This kind of spatial curriculum would also contribute to a broader vision of mathematical education as more inclusive and accessible. For instance, broader aims might include empowering all learners to solve problems of importance in their lives, to become critically thinking citizens and to effect change to make the world a better place, including social justice and eco-concerns. Spatial reasoning would contribute to such aims, for instance in developing proportional reasoning and understanding of graphical representations.

The mathematics curriculum is currently defined by dominant cultures, ignoring many children's interests, heritage and out of school mathematics experiences. It could be based on a broader view of mathematics, including:

- arithmetical, proportional, spatial, algebraic and statistical understanding.
- mathematical thinking and reasoning, including following lines of enquiry, fluency and problem solving.
- dispositions and habits of mind, such as curiosity, collaboration and perseverance.

Mathematics pedagogy is underpinned by views of how children learn: e.g. social constructivism, with children building understanding through networked associations from multi-sensory and emotional experiences, situated in social relationships. Current evidence from neuroscience endorses similar views (e.g. <sup>79</sup>). Pedagogical approaches therefore relate to views of mathematics as a subject and aims for mathematics learning, and to processes of learning and teaching.

These might include greater emphasis on:

- **mathematical thinking**, with teachers interested in what and how children think.

<sup>78</sup> Rose, J. (2009). *Independent Review of the Primary Curriculum: Final Report*. London: DCSF publications. <https://education-uk.org › 2009-IRPC-final-report>

<sup>79</sup> Rogers, C. & Thomas, M.S.C. (2023). *Educational Neuroscience*. London: Routledge. <https://doi.org/10.4324/9781003185642>

- **oracy and visualising**, including discussion, use of prediction, and representation of practical experiences.
- **inclusive participation**, with learners making sense collaboratively, in meaningful contexts, linking with out of school experiences and other subjects.

Spatial thinking therefore plays an important role in mathematics as a subject, and in pedagogical approaches. It supports:

- **geometry**, linking with geography, science, design and engineering.
- **representations** of problems, relationships and data e.g. diagrams, sketches and graphs.
- **effective learning** e.g. developing from practical experience with physical objects to abstract cognition using visualisation and prediction.
- **inclusivity**: making mathematics more enjoyable, sociable and multicultural, by allowing children to relate to mathematics in different ways, drawing on interests, aptitudes and out of school experiences.

A spatialised mathematics curriculum needs to include these elements.

It seems useful to divide Geometry into the two traditional areas of ‘space’ and ‘shape’. The main aspects identified from research are shown in the box below. These aspects relate closely to those identified by PISA’s ‘space and shape’ mathematical topic.<sup>80</sup>

## Aspects of spatial reasoning

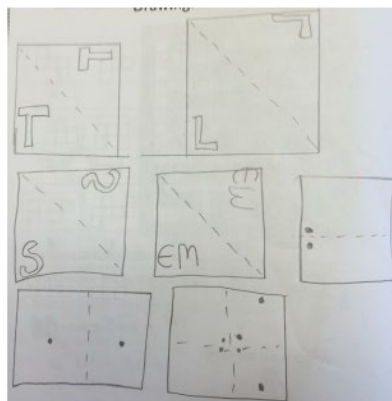
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<sup>80</sup> OECD, Organisation for Economic Co-operation and Development (2022). *PISA 2022 Mathematics Framework*. PISA, OECD Publishing, Paris. <https://pisa2022-maths.oecd.org/>

Space (spatial relations)	Shape (objects and properties)
<ul style="list-style-type: none"> <li>• <b>Position</b> – <i>Where?</i> In relation to one or two things, e.g. <i>next to, between, in front of, behind, relative to the viewer</i>, and using coordinates.</li> <li>• <b>Direction</b> – <i>Which way?</i> Moving around, e.g. <i>over, under, forwards/ backwards, left, right</i>.</li> <li>• <b>Dimensions</b> – Distance, e.g. <i>How far away?</i> Area, e.g. <i>near, in the middle</i>.</li> <li>• <b>Transformations</b> – Rotation (<i>turning</i>), translation (<i>sliding</i>) and reflection (<i>flipping</i>), e.g. moving a shape or jigsaw puzzle piece to fit or match.</li> <li>• <b>Perspective-taking</b> – Appearance from different viewpoints: <ul style="list-style-type: none"> <li>○ Visibility – <i>what</i> can be seen, e.g. hidden or partially visible.</li> <li>○ Position – <i>where</i> objects are in relation to each other, e.g. things behind each other appear to overlap.</li> <li>○ Orientation – <i>which way up?</i> e.g. <i>upside down, back to front, tipped over</i>.</li> <li>○ Appearance – e.g. <i>how</i> circles can look like ovals from certain viewpoints.</li> <li>○ Scaling – zooming in and out e.g. small-world play (toy farms), reading maps.</li> </ul> </li> <li>• <b>Navigation</b> – e.g. wayfinding, map-making and routes.</li> </ul>	<ul style="list-style-type: none"> <li>• <b>Identifying</b> – <i>What?</i> 2D and 3D objects (regular and irregular) such as cups, clothes, jigsaw pieces, leaves and clouds, e.g. <i>circle, rectangle, triangle, heart-shaped, cuboid, cone, ball, roof-shaped</i>.</li> <li>• <b>Properties</b> including: <ul style="list-style-type: none"> <li>○ Size, length, area, capacity and volume, e.g. <i>big, tall, wide; Will it fit in?</i></li> <li>○ Sides, faces, edges, lines, radius, e.g. <i>straight/ curved, wiggly, zig-zag; equal and unequal</i>.</li> <li>○ Corners, angles, e.g. <i>points, vertices, right angle, square corner, sharp, obtuse</i>.</li> <li>○ Symmetry – in 2D and 3D, reflective and rotational.</li> </ul> </li> <li>• <b>Composing and Decomposing</b> <ul style="list-style-type: none"> <li>○ Composing – fitting together 2D and 3D shapes, using interrelationships between properties, angles, e.g. with jigsaw puzzle pieces, pattern blocks, nesting containers and construction.</li> <li>○ Decomposing – disembedding (parts within wholes), cutting and folding, cross-sections, 2D and 3D, e.g. unfolding boxes to make nets and recomposing, <i>Which net would make a cube?</i></li> </ul> </li> <li>• <b>Scaling</b> – identifying the same item in different sizes, similarity, proportions, enlarging and shrinking, <i>doubling</i>.</li> </ul>

An extract from an example of a proposed spatial curriculum from birth to 11 is shown below, based on research indicating the progression in children’s learning in each of the aspects. However, since children’s spatial skills and understanding at any age vary greatly according to differences in experience as well as in development, this progression is presented in broad age groups. The terms, **early**, **middle** and **later** to refer to children from birth to 7 years, 7 to 9 years and 9 to 11 years, respectively. While the recommended experiences are based on research, they should not be regarded as targets or expectations. (See Appendix 1 for the full progression and numbered research references.)

TOPIC	Early	Middle	Later
<b>GEOMETRY: SPACE – learners should have opportunities to:</b>			
<b>Transformations</b>  References (see Appendix 1) 21, 8, 5	Use movement and rotation, <i>sliding, flipping, turning</i> . Develop simple 2D visualisation skills (mentally manipulating objects), e.g. imagining an object upside-down. Support using gesture and physical objects. Spatial patterns involving reflection and rotation.	Develop 2D and 3D visualisation skills for movement and rotations (mentally manipulating objects), <i>rotate, translate, reflect, clockwise, anticlockwise</i> , support by using prediction and checking. Reflect a pattern or arrangement over a horizontal or vertical line.	Predict the result of transformations, e.g. how more complex shapes (2D and 3D) will appear when rotated, the result of cutting and folding.  Reflect a pattern or arrangement over a diagonal line or in four quadrants.
<b>Perspective-taking</b>  References (see Appendix 1) 23, 20, 8, 30	Explore and predict what and how things look from different viewpoints	Visualise, draw and describe how objects look different from different viewpoints, e.g. overlapping objects, plan view  Interpret and predict what is different between two perspectives.	Visualise, draw, describe and interpret different viewpoints, re. visibility, position, orientation, appearance, scaling.



Transformations 9-10 years:  
Reflection over diagonal axis  
(Lowrie and Logan, 2023)<sup>83</sup>

Appropriate activities are suggested by interventions such as those in Canada and Australia.<sup>81–</sup>

<sup>83</sup> An example of a perspective-taking activity for the middle range is drawing and making a

<sup>81</sup> Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S. (2017). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a 32-week intervention. *Cognition and Instruction*, **35**(3): 236–264. <https://doi.org/10.1080/07370008.2017.1323902>

<sup>82</sup> Mulligan, J., Woolcott, G., Mitchelmore, M., Busatto, S., Jennifer, L., & Davis, B. (2020). Evaluating the impact of a Spatial Reasoning Mathematics Program (SRMP) intervention in the primary school. *Mathematics Education Research Journal*, **32**: 285–305. <https://doi.org/10.1007/s13394-020-00324-z>

<sup>83</sup> Lowrie, T. & Logan, T. (2023). Spatial visualization supports students' math: mechanisms for spatial transfer. *Journal of Intelligence*, **11**(6): 127. <https://doi.org/10.3390/jintelligence11060127>

model from different viewpoints. A transformation task for older children is to reflect images over a diagonal line (see above).

A learning trajectory with activities for children from birth to seven years has been developed by the Early Childhood Maths Group (see [ECMG Spatial Reasoning Toolkit](#)). A trajectory for children aged 7 to 11 is in progress. As this is a developing area, many approaches and activities remain to be evaluated: for instance, there is a wealth of IT resources including robots, virtual reality headsets, apps and programmes, which have a potentially significant role in developing visualisation, but have not yet been thoroughly researched.

These activities clearly link with other subjects. Mathematics is embedded in topics such as mapping the local environment, or contrasting the size of different creatures, which involve model making and drawing to scale. These kinds of activities engage young learners, while demonstrating the relevance of mathematics to different areas of life and culture. There are a wealth of primary school resources provided by subject organisations which include spatial reasoning applied to various topics, such as climate change (<https://geography.org.uk/online-teaching-resources/>) or the [scale of the universe](#) (Royal Academy of Engineering.) Many of these involve older children in interpreting data from internet sources, thinking spatially in more abstract ways.

#### **5.4. Relevance for mathematics as a whole**

Research also suggests that general mathematics pedagogy should be spatialised to include bodily enactment, physical manipulatives, spatial language and gesture, with children using sketching and visualisation to solve mathematics problems. Therefore, the teaching of number, algebra and statistics should include spatial representations and contexts, with a progression from practical experience to problems involving prediction, which require visualisation. This is not a progression in terms of age, with older children dealing only with abstract problems, but requires the selection of appropriate models and images for the aspect being taught at a particular age.<sup>84</sup> It is also not a linear progression from concrete to abstract within a topic, since children learn from checking, representing abstract expressions with manipulatives or drawings, moving between and connecting representations.<sup>85</sup> Below is an example of how spatial thinking is involved in learning aspects of number, algebra and statistics.

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<sup>84</sup> EEF, Education Endowment Foundation (2017). *Improving Mathematics in Key Stages 2 and 3*.

<https://educationendowmentfoundation.org.uk/education-evidence/guidance-reports/maths-ks-2-3>

<sup>85</sup> Griffiths, R., Gifford, S., & Back, J. (2017). *Using Manipulatives in the Foundations of Arithmetic: Literature Review*. Nuffield Foundation. <https://www.nuffieldfoundation.org/project/using-manipulatives-in-the-foundations-of-arithmetic-2>

TOPIC	Early	Middle	Later
<b>SPATIALISING THE CURRICULUM: NUMBER, ALGEBRA, STATISTICS</b>			
<b>Learners should have opportunities to use spatial representations and contexts, to visualise and predict</b>			
<b>Number:</b>  References (see Appendix 1) <b>3, 4, 25, 14</b>	Subitising, e.g. dice and Numicon patterns. Manipulatives and spatial images, e.g. rekenrek.  Number lines, including large-scale floor lines: whole numbers. Contexts: measuring length and capacity.	Multiplicative reasoning, e.g. arrays.  Number lines: identifying whole numbers and fractions, using number lines of different scales. Contexts: interpreting scales for length and capacity.	Proportional reasoning, including fractions and ratios, e.g. bar model.  Number lines: whole numbers, fractions, decimal fractions, negative numbers. Contexts: interpreting measuring scales involving decimals.
<b>Algebra</b> References (see Appendix 1) <b>17,12</b>	Repeating patterns, linear and circular. Algebraic relations with measures, e.g. Cuisenaire.	Simple growing patterns, e.g. triangular numbers.	Extending growing patterns to nth term.
<b>Statistics</b> References (see Appendix 1) <b>14, 25</b>	Interpreting block charts, pictograms, tally charts.	Interpreting bar charts, Carroll diagrams.	Interpreting line graphs, pie charts, scatter graphs.

For instance, a key model for understanding numbers is the number line, which very young children might experience as a number track to jump along outdoors, while older children might interpret a measuring scale on a jug in order to solve a practical problem.

### 5.5. Professional development implications

Teachers will need substantial support in understanding what spatial reasoning is, and how to teach it. Learning trajectories support teachers to develop children’s learning and to identify appropriate experiences and challenges.<sup>86</sup> Professional developmental should provide teachers with agency to use spatial representations and encourage children to think spatially across the mathematics curriculum.

## 6. Implications for the future

The mathematics curriculum needs revising and updating so that spatial reasoning is given more prominence for the benefit of children’s mathematics education and their futures. A new curriculum should include the aspects identified in the progression outline (see p25, Appendix 1).

In the broader context, mathematics is currently part of a high-stakes accountability system which tends to narrow the curriculum to what is measurable in tests, or observable through Ofsted’s lens.

<sup>86</sup> Clements, D.H. & Sarama, J. (2021). *Learning and Teaching Early Math: The Learning Trajectories Approach*. NY & London: Routledge

Mathematics in England is a discipline where white middle-class males dominate: the primary curriculum, in the way that it is presented, experienced and assessed, may contribute to this. A greater emphasis on spatial experiences and thinking might go some way to changing the image of mathematics and the way it is experienced by diverse children and teachers.

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## Appendix 1: Progression across age bands

Spatial thinking plays an important role in mathematics as a subject, and in pedagogical approaches. It supports:

- **geometry**, linking with geography, science, design and engineering.
- **representations** of problems, relationships and data, e.g. diagrams, sketches and graphs.
- **effective learning**, e.g. developing from practical experience with physical objects to abstract cognition using visualisation and prediction.
- **inclusivity**: making mathematics more enjoyable, sociable and multicultural, by allowing children to relate to mathematics in different ways, drawing on interests, aptitudes and out of school experiences.

A spatialised mathematics curriculum needs to include these elements and to present them in a coherent progression. However, since children's spatial skills and understanding at any age vary greatly according to differences in experience as well as in development, it seems more appropriate for a future curriculum to recommend experiences in broad age groups. Here we use the terms **early**, **middle** and **later** to refer to children from birth to 7 years, 7 to 9 years and 9 to 11 years, respectively. Recommendations are based on research but should not be regarded as targets or expectations. It seems useful to divide Geometry into the two traditional areas of 'space' and 'shape'. The main aspects identified from research are:

- **Space**: position, direction, dimensions, transformations, perspective-taking and navigation.
- **Shape**: identifying, properties, composing and decomposing and scaling.

TOPIC Reference no.	Early	Middle	Later
<b>GEOMETRY: SPACE – learners should have opportunities to:</b>			
<b>Position</b>  6, 11, 28	Respond to and use language of position – <i>in, on, under, up, down, next to, between, in front of, behind, opposite, overlapping</i> – including terms relative to the viewer, and supported by gestures.	Continue to use the language of position – <i>horizontal, vertical</i> – and supported by gestures.  Identify relative positions on a line and 2D positions within an area, using terms such as: <i>middle, midpoint, nearer, edge</i> , and simple coordinates.	Extend the language of position – <i>parallel, incline, decline, perpendicular, orientation</i> – and supported by gestures.  Identify and reason about position in 2D and 3D contexts, using more precise coordinates on maps, and in four quadrants.
<b>Direction</b>  1, 11, 16	Follow and give directions: <i>up, down, left, right, straight on, through, around</i> .  Make whole, half and quarter turns.	Continue to use the language of direction using appropriate language: <i>left, right, diagonal</i> .  Describe direction from the origin, e.g. left 3 and up five.  Identify turns that are more or less than 90 degrees, e.g. using Logo turtle.	Extend the language of direction: <i>north, south, east, west</i> .  Give and follow more complex sequences of directions, using greater accuracy to describe turns, e.g. using ICT to sequence instructions, to create shapes and patterns.
<b>Dimensions (measuring space)</b>  1, 2, 18	Use distance to identify the location of objects.  Compare and predict length/distance, volume/capacity, e.g. place and describe relative distances, <i>nearer to</i> .  Begin to use proportional language: <i>halfway, middle</i> .	Estimate distance between places in large-scale space.  Use representations to place things at approximately correct relative distances.  Continue to use the language of dimension.	Use more precise units, including decimals.  Understanding 2D representations of large-scale space, including heights and slopes: e.g. interpreting map contours
<b>Transformations</b>  21, 8, 5	Use movement and rotation: <i>sliding, flipping, turning</i> . Develop simple 2D visualisation skills (mentally manipulating objects), e.g. imagining an object upside-down. Support using gesture and physical objects. Spatial patterns involving reflection and rotation.	Develop 2D and 3D visualisation skills for movement and rotations (mentally manipulating objects) – <i>rotate, translate, reflect, clockwise, anticlockwise</i> – support by using prediction and checking. Reflect a pattern or arrangement over a horizontal or vertical line, progressing to a diagonal line.	Predict the result of transformations, e.g. how more complex shapes (2D and 3D) will appear when rotated, the result of cutting and folding.  Reflect a pattern or arrangement over a diagonal line or in four quadrants.

<b>Perspective-taking</b>  23, 20, 8, 30	Explore and predict what and how things look from different viewpoints	Visualise, draw and describe how objects look different from different viewpoints, e.g. overlapping objects, plan view. Interpret and predict what is different between two perspectives.	Visualise, draw, describe and interpret different viewpoints, re. visibility, position, orientation, appearance, scaling.
<b>Navigation</b>  29, 19, 26, 18	Navigate and remember simple fixed routes, using landmarks.  Interpret and make simple 2D and 3D maps of familiar environments.	Interpret and draw maps.  Plan routes using nearby landmarks.  Begin to determine simple shortcuts.	Orienteer using maps of unfamiliar environments, rotating to follow directions.  Begin to plan and evaluate alternative routes.  Begin to make use of distant landmarks as well as nearby landmarks.
<b>TOPIC</b>	<b>Early</b>	<b>Middle</b>	<b>Later</b>
<b>GEOMETRY: SHAPE – learners should have opportunities to:</b>			
<b>Identifying shapes</b>  27, 31	Identify similar, varied, extreme and non-examples of the same shape in different orientations, by gesture and using common and increasingly precise names (e.g. triangle, rectangle, rhombus, cuboid, cylinder, sphere).	Define and classify geometric and non-geometric 2D and 3D shapes (including regular and irregular shapes and everyday objects, e.g. platonic solids and prisms, clouds and leaves).	Define, classify, draw and visualise geometric 2D and 3D shapes (e.g. using isometric paper).
<b>Properties</b>  28	Explore properties physically and describe, using gesture, informal language, analogies and mathematical terms (e.g. <i>straight, curved, points, zigzag, roof-shaped, faces, right angles</i> ).	Discriminate shapes by properties including angle, perimeter, area and topological properties.	Discriminate shape properties, including symmetry and angle measurement
<b>Composing and decomposing</b>  1, 21, 22	Create shapes, models, structures and arrangements of increasing complexity, solving puzzles, planning and predicting by visualising. Begin to predict folds, nets and cross-sections.  Compose and decompose shapes, knowing how shapes combine to make other shapes (e.g. triangles making a rectangle) and identifying shapes within shapes (decomposing).	Create more complex models, constructions and arrangements. Visualise and predict folds, nets and cross-sections of increasing complexity, checking using physical objects.  Find shapes within complex arrangements, then combinations of lines within overlapping shapes (disembedding).	Create more complex models, constructions and arrangements. Continue to visualise and predict folds, nets and cross-sections of increasing complexity, reducing support from using physical objects.

<b>Scaling</b>	Recognize scale models and their relative sizes, e.g. larger and smaller animals of the same scale.	Identify the same item in different sizes (similarity), considering proportions, enlarging and shrinking, doubling and halving.	Make scale drawings and models.
10, 18	Predict what fits and what doesn't fit (e.g. which teddy will fit in the box).	Determine the same location on two maps at different scales.	
<b>TOPIC</b>	<b>Early</b>	<b>Middle</b>	<b>Later</b>
<b>SPATIALISING THE CURRICULUM: NUMBER, ALGEBRA, STATISTICS</b>			
<b>Learners should have opportunities to use spatial representations and contexts, in order to visualise and predict</b>			
<b>Number:</b>	Subitising, e.g. dice and Numicon patterns. Manipulatives and spatial images e.g. rekenrek.	Multiplicative reasoning e.g. arrays.	Proportional reasoning, including fractions and ratios, e.g. bar model.
3, 4, 25, 14	Number lines, including large scale floor lines: whole numbers. Contexts: measuring length and capacity.	Number lines: identifying whole numbers and fractions, using number lines of different scales. Contexts: interpreting scales for length and capacity.	Number lines: whole numbers, fractions, decimal fractions, negative numbers. Contexts: interpreting measuring scales involving decimals.
<b>Algebra</b>	Repeating patterns, linear and circular. Algebraic relations with measures, e.g. Cuisenaire.	Simple growing patterns, e.g. triangular numbers.	Extending growing patterns to nth term.
17,12			
<b>Statistics</b>	Interpreting block charts, pictograms, tally charts.	Interpreting bar charts, Carroll diagrams.	Interpreting line graphs, pie charts, scatter graphs.
14, 25			

**Note:** *Early*, *middle* and *later* age ranges refer to children from birth to 7, 7 to 9 and 9 to 11, respectively. However, children's previous experiences will mediate what is appropriate for individuals in different contexts.

## Reference key for Appendix 1 and table extract p20

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