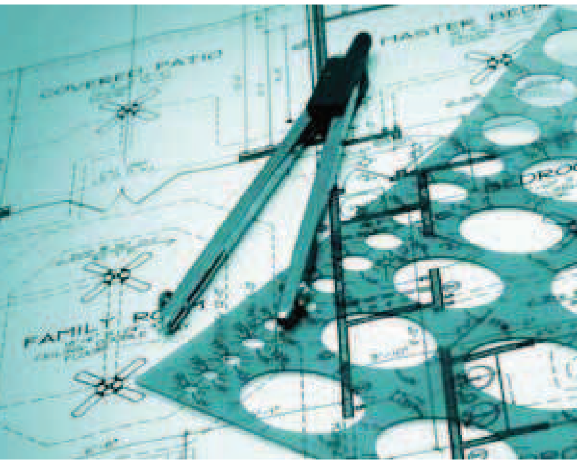
ACME

Advisory Committee on Mathematics Education



Mathematical Needs

Mathematics in the workplace and in Higher Education

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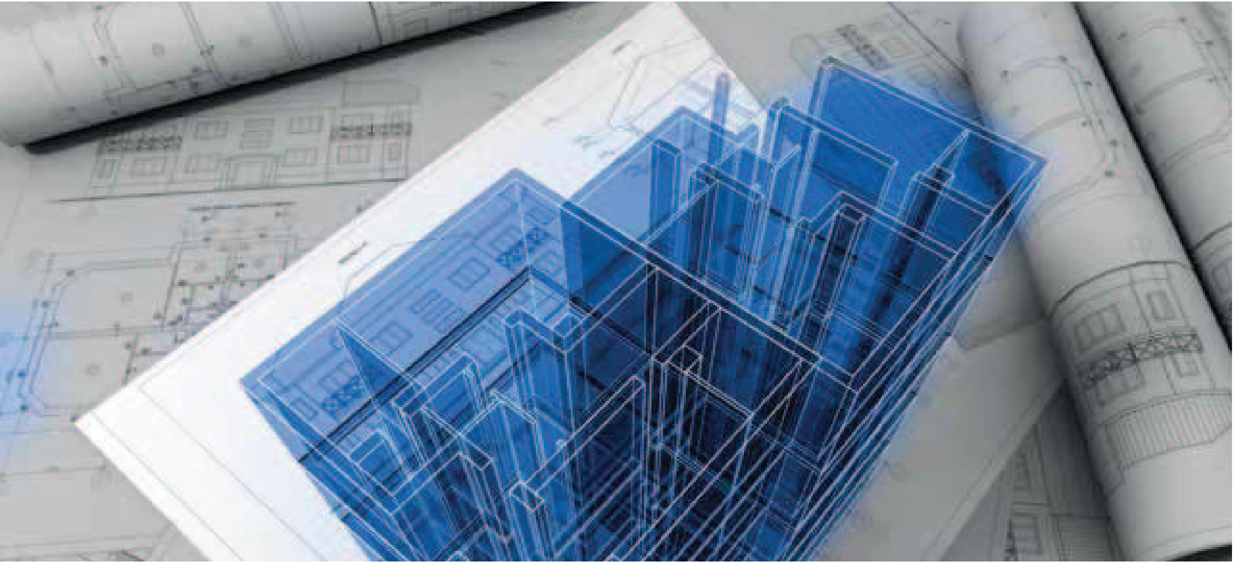
Contents

A Note on Terminology

This report is about mathematics and how it is applied. However, a significant part of the testimony that we have received concerns issues related to numeracy and to statistics. We have included a glossary of the terms that we have employed in Appendix 4. In this report, we use the unqualified term mathematics to mean mathematics in general, and so to include both numeracy and statistics. Where we feel a need to identify issues that relate specifically to statistics this is made clear in the text. The case of 'numeracy' is more complicated as different professional groups in education use this word in various ways. We discuss this in Appendix 4. We recognize that in many official reports 'numeracy' is used to indicate lower levels of mathematics. However, skill with numbers is needed at all levels, and in many complex contexts, so we have chosen to use the terms 'basic numeracy' or 'basic mathematics' for more elementary uses of mathematics.



1. Executive Summary

This is one of a pair of reports from the Advisory Committee on Mathematics Education (ACME, acme-uk.org)[1] outlining the country's mathematical needs. In this report, mathematical needs are described from the perspective of higher education and employers; in the other, the focus is on the needs of students learning the subject. These reports are designed to provide an evidence base for future policy decisions about the national provision of mathematics.

The research for this report led us to interview many people, both in universities and across the sectors of employment, as well as carrying out a review of existing literature and data sources. The overwhelming message, from virtually all respondents, is that **we need more young people to know more mathematics and to be confident in using it.** All those leaving school or college should have confidence in mathematics and an understanding of it that allows them to use it immediately and to participate effectively in continuing education and training.

In these times of rapid change the demand for improved mathematical skills and understanding is growing. Two important reasons for this are:

- **The quantitative demands of almost all university courses are increasing**; even subjects like history, which traditionally had involved no mathematics, now recognize the importance of statistics.
- **In the workforce there is a steady shift away from manual and low-skill jobs** towards those requiring higher levels of management expertise and problem-solving skills, many of which are mathematical in nature.

These are global trends that the UK cannot avoid.

In the Organisation for Economic Co-operation and Development (OECD) countries, and other developed countries, the proportion of the population studying mathematics post-16 is significantly higher than in the UK. These countries foster the development of mathematics learning because of the economic importance of having a strong mathematical skills base in undergraduate study, in work and in research and development. The UK can only ignore this contrast at its peril.

Each year a cohort of about 700,000 young people passes through the UK's education system. Many of these young people study no mathematics beyond the age of 16, the current minimum school leaving age in the UK. Of these, nearly half will not have achieved even a minimum C-grade pass in GCSE mathematics. A minority of 18-year-olds progress from secondary education to study mathematics or highly related mathematical disciplines at university. Many more, against expectations, find that they need mathematics in the workplace or in their chosen course of study.

A major problem in England, together with Wales and Northern Ireland, is the two-year gap for most young people between the end of GCSE and the start of university or employment, during which the large majority do no mathematics. This is the most striking and obvious difference between the mathematics provision here and that in other comparable countries. It is not just a case of students missing out on two years of learning mathematics, serious though that is, but of their arriving at the next stage of their lives having forgotten much of what they did know.

Our research revealed some of the effects of this deficit. **We estimate that of those entering higher education in any year, some 330,000 would benefit from recent experience of studying some mathematics (including statistics) at a level beyond GCSE, but fewer than 125,000 have done so.** This places those responsible for many university courses in an impossible position. They cannot require an appropriate level of mathematics of their applicants and hope to fill their places, and in many cases they are unable to design courses with the level of quantitative demand that would be appropriate for their disciplines.

Employers emphasized the importance of people having studied mathematics at a higher level than they will actually use. That provides them with the confidence and versatility to use mathematics in the many unfamiliar situations that occur at work. A frequently heard comment was that too many young people have only learned to do the sort of questions that are set on GCSE papers. We found a remarkable degree of unanimity among employers as to what they do want, and this is captured in this report.

[1] The research for this report has been funded by the Nuffield Foundation and the Clothworkers Foundation.

Trying to teach the whole cohort AS and A-level mathematics would be doomed to failure. Additional, new courses and qualifications are needed for those who currently opt out of mathematics. Such courses must be relevant to those taking them and take into account their prior attainment. This report, and the extensive underlying research evidence, provides new insights into what this might mean.

It is sometimes suggested that all that is needed is to teach students how to carry out the particular calculations that are common in their intended field of employment. Many of the employers we interviewed provided a different perspective, recognizing that mathematics is a subject of intellectual power and that the best interests of their companies would not be served by an education system restricting young people to a diet of the particular techniques they were likely to use in their day-to-day work. Even though their major engagement with mathematics is likely to relate to its application, some will be inspired to study to a high level for its own sake, while others will glimpse the beauty and abstraction of the subject. Happily, there is no conflict between formal mathematics and its use in a widening range of situations. Instead there is an increasing awareness, which the research for this report reinforces, that **developing a sound understanding of key mathematical ideas is an essential element in a good modern education.** This requires extensive experience of working with the basic mathematical concepts that underpin its range of applications.

It is sometimes argued that the advent of computers has reduced the need for people to be able to do mathematics. Nothing could be further from the truth. Off-the-shelf and purpose-designed computer software packages are creating ever more data sets, statistics and graphs. Working with mathematical models, which people need to be able to understand, interpret, interrogate and use advantageously, is becoming commonplace. The use of quantitative data is now omnipresent and informs workplace practice.

The focus of the higher education part of the report is the mathematical requirements of a wide range of courses, many of which (as will become clear on reading this report) include significant elements of mathematics. However, the report would be incomplete without some mention of the needs of those going on to study highly mathematical disciplines such as mathematics itself, physics and engineering. For all these students, A-level Mathematics is essential and in some universities, they will also require Further Mathematics. The mathematical needs of those disciplines are considered in this report.

2. Recommendations and Commentary

POLICY

Recommendation 1

Policy on mathematics post-16 should ensure that a large majority of young people continue with some form of mathematics post-16.

Mathematics dropped at age 16 is easily forgotten, and skills are not consolidated.

Recent research evidence shows that many other countries ensure that a much higher proportion of their population studies at least some mathematics beyond the age of 16.

The research in this report indicates why it is important for more students to continue studying mathematics beyond GCSE.

A particular group who will benefit is that of the many primary school teachers whose highest qualification in mathematics is currently GCSE Grade C, obtained many years before they first enter the classroom.

Recommendation 2

A wider curriculum and provision than exists at present should be developed in order to ensure that all young people are well placed to benefit from their studies in mathematics.

While it is very desirable to increase the number of students taking AS and A-levels in mathematics and further mathematics, these students will only ever be in a minority and attention must also be directed to the rest of the cohort.

THE NATIONAL CURRICULUM (UP TO AGE 16)

Recommendation 3

The 2011 National Curriculum Review should ensure that in the new curriculum greater attention is given both to essential mathematical techniques and the application of mathematics.

Significant numbers of young people in higher education (HE) and in the workplace have problems with their fluency in standard techniques. Both employers and HE lecturers say that many recruits often make mistakes of a basic nature, and in addition cannot apply the mathematics that they have learned.

Recommendation 4

The 2011 National Curriculum Review should ensure that in the new curriculum students are familiarized from an early stage of their education with the concept of a mathematical model. Mathematical models should be drawn from a wide range of areas.

The development and subsequent application of mathematical models in the workplace is pervasive, and an early exposure to the modelling process in mathematical learning is essential.

THE NATIONAL CURRICULUM (UP TO AGE 16) *continued*

Recommendation	Rationale
Recommendation 5 **Students need to experience the power of computer software when applied to problems that can be formulated mathematically.**	The extensive use of information and communications technology (ICT) in the workplace has changed not only the way that work is done, but also the work itself. The availability of increased computational power means that analyses that were previously unavailable because they would have taken too much time to produce are now carried out routinely. It has been suggested that the pervasive presence of computers to carry out calculations has reduced the need for mathematics in the workplace. In fact, the opposite is true. Because more sophisticated analyses are available at the click of a mouse, the workforce needs to be more mathematically competent in order to understand and interpret the information produced by these analyses.

PROVISION POST-16

Recommendation	Rationale
Recommendation 6 **A new post-16 mathematics provision should inform and be informed by the reformed National Curriculum.**	A holistic approach to the education system is essential. We should aim for a seamless development of students' confidence, knowledge and skills across the transition at age 16.
Recommendation 7 **Additional courses should be developed for the post-16 cohort, so as to extend the current provision to cover the full range of students both in terms of their career aspirations and also their prior attainment in mathematics. The major elements in such new courses should include statistics, problem-solving and working with mathematical models. Sufficient time also needs to be allocated for study and assimilation of fundamental concepts.**	The evidence from this research indicates the present lack of statistics, mathematical modelling and problem-solving skills.
Recommendation 8 **The new post-16 courses should include statistics beyond the descriptive methods of GCSE to meet the needs of the large number of students who progress to a range of courses such as those in social and life sciences.**	Individuals are often underprepared mathematically for the statistics/quantitative methods components of degrees in social and biological sciences. Statistics provision post-16 should include inference, experimental design, probability, risk and the use of statistics to aid decision-making.
Recommendation 9 **The value of being able to communicate mathematics should be given more prominence as this is an essential skill in employment and in HE.**	In the workplace effective application of mathematics must be allied to clear communication of the results to the appropriate audience. Many employers are concerned that some otherwise mathematically capable young people are not able to communicate their understanding to their line manager or colleagues.
Recommendation 10 **There should be an emphasis on building students' confidence and their ability to use mathematics in a range of familiar and unfamiliar contexts.**	Many employers are concerned that young people are unable to apply the mathematics they know. To foster this, mathematics should be studied to a higher level than that at which it is likely to be used.

ASSESSMENT

Recommendation	Rationale
Recommendation 11 **New methodologies should be developed to ensure that the required learning outcomes are adequately assessed, at all stages.**	The range of assessment instruments is currently inadequate to foster the mathematical and communication skills needed to implement these recommendations. Employers and HE staff say that there is 'too much teaching to the test', and this is detrimental to the development of the required skills. This distortion is reinforced by the use of assessment results as a measure of institutional accountability.

TEACHER EDUCATION AND CONTINUING PROFESSIONAL DEVELOPMENT

Recommendation	Rationale
Recommendation 12 **Initial teacher education (ITE) and continuing professional development (CPD) should be expanded in order to ensure a sufficient supply of competent teachers of mathematics to implement the recommendations of this report.**	We need many more effective teachers of mathematics if we are to increase the size of the cohort studying mathematics post-16. These will not be found solely from mathematics graduates. We note that government agencies and others have been developing programmes to enable individuals from other disciplines to become effective teachers of mathematics. It is essential that these programmes are further supported and developed.
Recommendation 13 **The education of teachers of mathematics should reflect the impact of computers and other forms of ICT on how mathematics is used in employment and HE.**	Programmes of ITE and CPD for teachers of mathematics should support the use of ICT within their teaching practice, covering a range of mathematical contexts and applications.

INFORMATION, ADVICE AND GUIDANCE

Recommendation	Rationale
Recommendation 14 **Universities should make clear the level and extent of mathematics encountered within each of their degree programmes.**	At present, many universities do not indicate the mathematics to be encountered within their degree programmes in a variety of subjects. As a result, many 16-year-olds decide not to continue mathematics post-GCSE, not realizing that it would be much to their advantage to do so; similarly, those who do take mathematics (e.g. at A-level) are often unaware of the options that would best suit them.
Recommendation 15 **Universities should state explicitly where they have preferences for qualifications in mathematics.**	The ability of students to make good choices for the mathematics they do is compromised by the lack of information from universities about the mathematics content of their courses.
Recommendation 16 **Teachers should be provided with information about the wider uses and value of particular mathematical ideas. Teachers need to know about the mathematical needs of employers and what is desirable on courses in HE; they must be encouraged to frequently include non-routine and unfamiliar situations, and opportunities for reasoning, in their teaching.**	This should be covered in teachers' CPD. There is also a need for this information to be with senior management teams so that students can be advised appropriately.

3. Mathematics in the UK Compared to Other Developed Nations

3.1 Take-up of mathematics post-16 in other countries

Recent research carried out by a team from King's College, London, supported by the Nuffield Foundation (Hodgen et al. 2010), has revealed that remarkably few people in England, Wales and Northern Ireland study mathematics, whether at an advanced or less advanced level after the age of 16, compared to other countries. The study reviewed the mathematics followed by the post-16 cohort in each of 24 countries or states (with each of the UK nations being considered separately). Based on data from each country, it was possible to categorize the level of mathematics studied and the degree of participation at the two levels considered ('any mathematics' and 'advanced mathematics'). The results shown in Table 1 are based on this Nuffield report.

England, Wales and Northern Ireland are at the bottom of the table. For England, Wales and Northern Ireland, less than 19 per cent of the age cohort study any mathematics at all beyond 16, whereas in eighteen of the countries considered more than half of the age cohort follows some mathematics, and in ten of these 95 per cent or more study some mathematics. Less than 15 per cent of the cohort in England, Wales and Northern Ireland study 'advanced mathematics', whereas in four Pacific Rim countries the proportion of those studying advanced mathematics is over 30 per cent. In a further nine countries, including Scotland, the proportion studying advanced mathematics is between 15 and 30 per cent. The research underpinning this report shows that the low participation in England, Wales and Northern Ireland is creating major problems in higher education and employment, and should be a source of serious concern to the nation's leaders.

Table 1
Students taking mathematics post-16

	Any mathematics	Advanced mathematics
Japan	All	High
Korea	All	High
Taiwan	All	High
Estonia	All	Medium
Finland	All	Medium
Germany	All	Medium
New Zealand	All	Medium
Sweden	All	Medium
Russia	All	Low
Czech Republic	All	-
France	Most	Medium
Massachusetts (USA)	Most	Medium
Ireland	Most	Low
British Columbia (Canada)	Most	-
Hungary	Most	-
Singapore	Many	High
New South Wales (Australia)	Many	Medium
Netherlands	Many	Low
Hong Kong	Some	Medium
Scotland	Some	Medium
Spain	Some	Low
England	Few	Low
Northern Ireland	Few	Low
Wales	Few	Low

Source: The Nuffield Foundation

Any mathematics	5-20%	20-50%	50-80%	80-95%	95-100%
Advanced mathematics	0-15%		15-30%		30-100%

Data on participation in advanced mathematics were insufficient in Czech Republic, British Columbia (Canada) and Hungary.

3.2 Patterns of education in the post-16 cohort in England

In order to understand the context within which mathematics is studied post-16, we have reviewed the data available from UCAS, JCQ and ONS[2] about the educational activities undertaken by 16–18-year-olds in England. The data show that three-quarters of the age 17 cohort[3] are now in full-time education (see Chart 1 below) and, of those in full-time education, 57 per cent are studying A-levels (see Chart 2 below). There are some 72,500 (about 11%) taking mathematics at A-level out of the total age cohort of about 660,000. The available evidence about the mathematics being taken by those not studying it at A-level suggests that the numbers progressing to some mathematics at level 3 are much lower than the numbers taking A-level mathematics. There are some individuals re-sitting level 2 GCSE mathematics, usually repeating the programmes with which they have already had little success.

The data on this cohort are summarized in Charts 1 and 2[4].

Chart 1
Education/employment data for total age 17 cohort in England, 2009 (666,700 individuals)

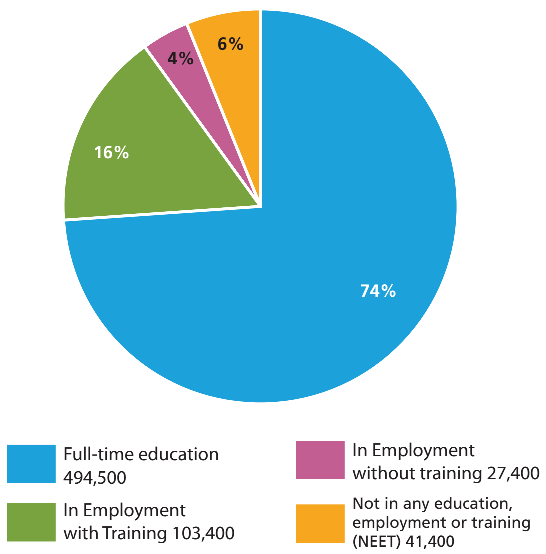


Chart 1 illustrates data on the whole age 17 cohort for 2009. Three-quarters of the cohort are in full-time education. One-fifth are in employment, and the majority of those receive training. The category of NEET ('not in any education, employment or training') represents 6 per cent of the cohort.

Chart 2
Breakdown of data for age 17 cohort in full-time education in England, 2009 (494,500 individuals)

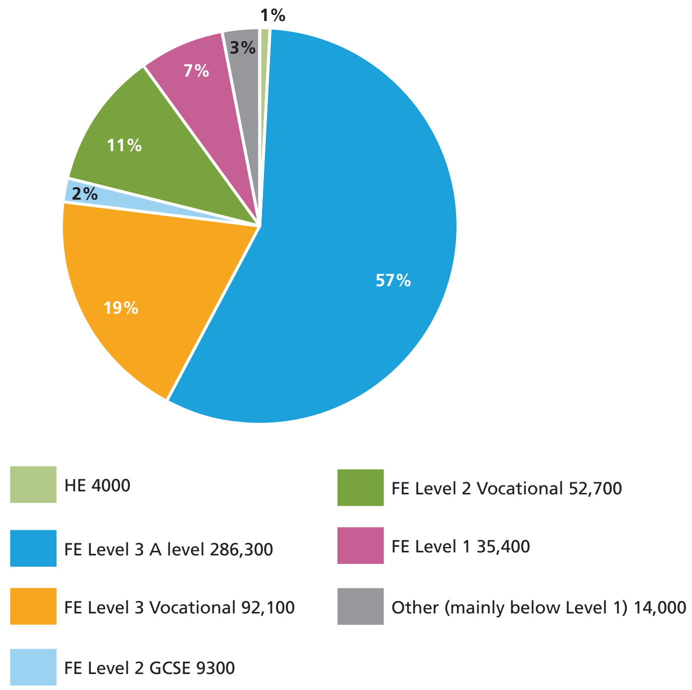


Chart 2 then analyses what those in full-time education are doing. A very small proportion have already started on higher education (HE) programmes. All programmes below HE are classed as further education (FE) in these data. Over half of those in full-time education are doing A-levels; nearly a fifth of them are doing vocational courses at level 3. Most of the remainder are following vocational courses at levels 1 or 2, with a small group taking a programme of GCSE resits. Information on the mathematics taken by those doing A-levels is readily available; it is not so easily found for those following vocational courses[5]. Many require the Key Skills qualification in Application of Number at level 2 (soon to be replaced by Functional Mathematics at level 2), but if candidates already have a C at GCSE mathematics they are exempted from this.

Of the 286,300 individuals taking A-levels in England, 72,750 are taking an A-level in mathematics, of whom approximately 10,500 are also taking further mathematics[6]. This also means that nearly 214,000 A-level candidates are not taking a mathematics A-level.

The majority of those taking A-levels progress to higher education. The programmes they follow are dependent on their results at A-level, in terms of grades obtained and subjects studied. The research we report on in greater detail in Chapter 4 describes how many university courses accept individuals with no more than a Grade C at GCSE in mathematics, even though the university course may make significant mathematical demands, which many students have difficulty in meeting.

2 Respectively: Universities and Colleges Admissions Service, Joint Council for Qualifications and Office of National Statistics.
3 The 'age 17 cohort', are those who are 17 on 1 September – and who attain their 18th birthday during the year under study. Those who are in school are in Year 13.
4 Department for Education, Statistical First Release 18/2010; Participation in Education, Training and Employment by 16–18 Year Olds in England, 22 June 2010.
5 The Wolf Report on Vocational Education, commissioned by the Department for Education, offers further information about mathematics in vocational education.
6 The Royal Society '***state of the nation***' report, *Preparing for the transfer from school and college science and mathematics education to UK STEM higher education* contains more information about the combinations of science A-levels and Scottish Highers and Advanced Highers completed by students taking post-16 mathematics in the UK.

3.3 Take-up of mathematics post-16 in the UK

A key figure is the number of students who leave school or college each year having taken some form of mathematics beyond GCSE (ie at level 3) during the previous two years. An estimate of this is given in Table 2 below; it is based on the figures for 2009. However, it should be noted that, for various reasons, precise data are not available and so the figures have been rounded. Even if exact data were available, they would change from one year to the next. Much of the uncertainty is caused by the danger of double-counting students; the figures in the table refer to individuals who do not appear elsewhere in the table.

Although the ACME remit applies to England only, in this case we have chosen to include data from Northern Ireland, Scotland and Wales, rather than bring in further inaccuracy by trying to separate them out. In Section 4 of this report, the figures obtained will be compared to estimates of the demand from higher education and that too is estimated for the whole of the UK; so we will be comparing like with like.

The figures for AS and A-level Mathematics include those for the equivalent qualifications in Use of Mathematics, Pure Mathematics and Statistics, but not those for Further Mathematics. The final column of the table indicates the size of the course; 1 unit is 60 guided learning hours.

Table 2
Mathematics courses taken by students leaving school/college in the UK, 2009[7]

Course	Students[8]	Course size (number of units)
A-level further mathematics + A-level mathematics	10,500	12
AS further mathematics + A-level mathematics	2500	9
A-level mathematics only	59,500	6
AS mathematics only	14,000	3
International Baccalaureate higher and further mathematics	20	9.75
International Baccalaureate mathematics higher level	1000	6
International Baccalaureate standard level	1500	3.75
International Baccalaureate mathematical studies (SL)	2000	3.75
Free Standing Mathematics Qualification	5000	1
Level 3 Vocational Mathematics Unit	8500[9]	1
Scottish Advanced Higher + Scottish Higher	3000	6
Scottish Applied Higher + Scottish Higher	500	6
Scottish Higher only	16,500	3
Total number of students	124,520	
Average number of units per student	5.23	

Thus the total number of students taking some level 3 mathematics between GCSE and the start of higher education or employment is about 125,000. This is the supply of home-grown students with some mathematical background beyond GCSE available to Higher Education. The majority of these do a substantial amount of mathematics (6 units or more).

7 Data sources are JCQ, SQA and FMSP (for vocational courses).

8 At AS and A-level, a new regulation took effect in 2009 allowing students to apply for certification on more than one occasion, with the result that some may be double-counted. There is evidence (MEI 2009) that this does not happen to any significant extent with A-level, and so the overall figure (ie the total of the first three rows) is reasonably reliable. However, the changed regulation makes it impossible to know with any accuracy the number taking only AS mathematics. The estimate is based on historical continuation rates of 83–85% from AS to A-level mathematics.

9 Data on take up of optional mathematics units within vocational courses has proved elusive, but the figure quoted is our best estimate based on discussions with relevant Awarding Organisations.

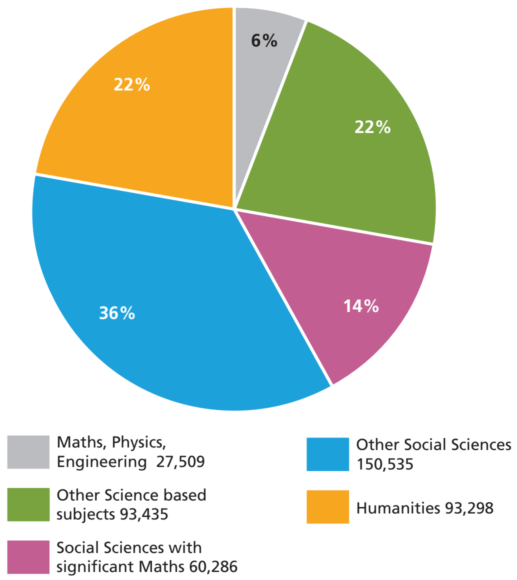
4. Mathematics in Higher Education

The mathematics studied by individuals in the Year 13 cohort will help to prepare them for their courses in higher education. Depending on the subject followed in higher education there will be a greater or lesser need to have mastered some mathematics. In order to investigate what mathematics individuals are likely to encounter in higher education courses a sample of subjects at a sample of universities were investigated. Course details were accessed through the internet.

4.1 Methodology

In order to categorize the subjects taken in university by likely level and perception of difficulty of the mathematics required on the university course, data from UCAS on those progressing to degree-level programmes in 2009 have been analysed, with the results as shown in Chart 3 below[10]:

Chart 3
University acceptances for all UK domiciled students progressing to degree-level programmes in the UK, 2009 (425,063 individuals)[11]



In Chart 3, we have separated those subjects that obviously require a significant amount of mathematics from those with less of a mathematical requirement. To a certain extent such divisions are arbitrary, but the research into the mathematics content of degrees that we report in Section 4.3 below indicates that several 'social sciences' courses (such as economics and finance) make significant mathematical demands of their students. We have also included nursing in this group because of the critical importance of mathematical competence in the work of nurses. Sports science is also part of this group. 'Other social sciences' also require some mathematics – especially statistics. These have been classed separately from humanities subjects, many of which make limited mathematical demands, though even here subjects like philosophy, law and design can have significant mathematics components.

Chart 3 shows the total numbers recruited onto HE courses in 2009. The data held by UCAS on university acceptances make it possible to carry out an analysis of those progressing to HE who have studied A-level. Appendix 5 summarizes the results of this analysis and compares the proportions of those with A-levels accepted onto different subjects with the total number accepted.[12] The proportions in the different categories are very similar.

Higher education institutions have organized themselves into four different groups: (i) Russell, (ii) 1994, (iii) University Alliance and (iv) Million+. There is a fifth category of those that are unaffiliated. In order to have a sample with sufficient breadth within each of the three types of subject listed it was decided to survey about ten different subjects in three universities in each of the university groupings listed above. In the event, just over 170 surveys were collected.

10 The subject groups used for this chart comprised the following:
Mathematics, physics, engineering: Mathematics, statistics, physics, engineering.
Other science-based subjects: Chemistry, materials science, astronomy, geology, physical geography, architecture, building and planning, computer science, information systems, medicine, pharmacology, toxicology and pharmacy, nutrition, medical technology, biology, botany, zoology, genetics, microbiology, veterinary science, agriculture.
Social sciences with significant mathematics: Psychology, sports science, nursing, economics, finance.
Other social sciences: Politics, sociology, social policy, social work, anthropology, human and social geography, law, business studies, management studies, human resource management, hospitality, leisure, tourism and transport, mass communications and documentation, education.
Humanities: Linguistics, classics, European and non-European languages and literature, history and philosophical studies, creative arts and design.

11 Note that the UCAS data used in Chart 3 apply to the whole of the UK, whereas the data on educational routes in Charts 1 and 2 only apply to England.

12 We are indebted to Sir Brian Follett of the STEM Forum for providing us with the data on A-level acceptances, and to Hannah D'Ambrosio of UCAS for help in interpreting the UCAS data.

The universities selected were:

- *Russell group*: the University of Cambridge, the University of Manchester, University College London (UCL).
- *1994 group*: the University of Lancaster, the University of Leicester, the University of York.
- *University Alliance group*: De Montfort University, University of Gloucestershire, the Manchester Metropolitan University.
- *Million+ group*: the University of Bolton, the University of Sunderland, the University of East London.
- *Unaffiliated group*: Aston University, the University of Kent, the University of Brighton.

The information gathered has enabled us to identify the numbers who are going to university in each of the five broad subject groups.

The survey involved extracting information from the universities' websites on (i) the mathematical requirements for entry to the course and (ii) the mathematical demands of the course. We report on these two different aspects of the research in Sections 4.2 and 4.3. below.

Information on the entry requirements was easy to obtain, but the information on the mathematics required during courses, although easily available in some cases, often required further investigation. Therefore, this was supplemented by a small number of interviews with staff in universities involved in providing mathematics support for students, including visits to the Sigma mathematical support centres in Loughborough and Coventry universities. The remedial support in mathematics offered by universities is discussed in Section 4.3.

4.2 University statements of entry requirements

There are significant differences between the mathematical prerequisite requirements made by different universities for the same subjects. The mathematical attainment required for entry to university for a given subject can vary enormously.

Generally, it is universities in the Russell group who tend to ask for higher levels of mathematics attainment. This parallels a distinction that is made by some in the university sector between 'selecting' and 'recruiting' institutions. For a 'selecting' institution, mathematics requirements can be useful in filtering the applications, whereas for a 'recruiting' institution the inclusion of mathematical requirements can reduce the number of applicants to unsustainably low levels.

From Chart 3 it is possible to infer that some 180,000 of those accepted into university will encounter a significant amount of mathematics on their degree courses. An additional 150,000 students in the social sciences will also encounter some mathematics on their courses, making a total of 330,000. University entry requirements do not reflect this need. The reason for this is not negligence on the part of universities but the complete mismatch betwen the demand figure for 330,000 students with mathematical skills and the supply figure of 125,000. This is a direct consequence of the situation illustrated in Table 1, with very few students continuing mathematics beyond the age of 16. Universities have no choice but to work with the students who come to them from schools and colleges.

4.3 University course design

The information available from the courses selected was reviewed in order to identify the mathematical demands made of the students while on the course. The level of detail available about the mathematics pursued on these courses varied significantly. Follow-up emails and discussions helped to gain more information in some cases, but in others only minimal information was available.

Mathematics modules are sometimes not titled as such. For example, in one university there is a 'Foundations of Computing' course that students on computing degrees take in the first year, which consists of the mathematics that students need to understand in order to complete the courses successfully. The context in which this is delivered is that some 70 per cent of the students do not have mathematics beyond GCSE, and for some of these the GCSE mathematics was not at the higher tier. The tutor of this course estimates that if a stronger mathematics background could be assumed for the cohort then this module could be shortened, making room for an additional computing topic, and there would also be beneficial effects on the ordering of other modules on the course.

Another example is an environmental geography course for which there is no stated mathematics entry requirement. However, the course involves the following topics, much of the content of which lies within AS or A-level mathematics:

- Appreciate the importance of checking physical units for consistency when rearranging.
- Use exponentials and logarithms to model natural processes.
- Apply curved functional forms to environmental, economic and social relationships (curved functional forms will include: quadratic and cubic polynomials, reciprocal functions, exponential functions, power law functions and logarithmic functions).
- Plot and sketch polynomial equations up to cubic form.
- Understand the meaning of a derivative of a mathematical function.
- Differentiate polynomial, exponential and logarithmic functions.
- Use differentiation as a means of finding maxima and minima of polynomial functions.
- Set up and solve optimization problems, given a verbal explanation of an optimization requirement.

Many courses in the social sciences include modules on statistical methods or research methods, which can involve substantial amounts of mathematics. However, Professor John MacInnes has found that for several social science subjects a separation is developing between those (ostensibly in the same discipline) who are comfortable with using quantitative methods in their research and those who prefer not to use quantitative methods (MacInnes 2009).

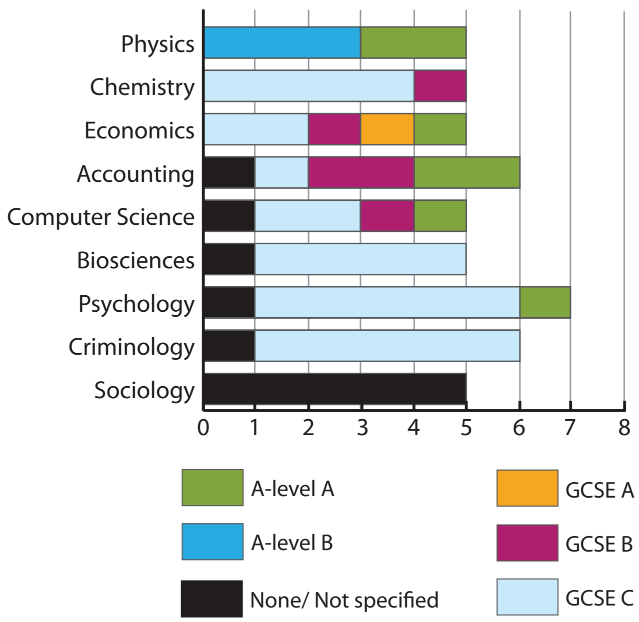
All that is sometimes available in course information literature is the title of a module involving mathematics, such as 'Understanding and Using Statistical Information' (this is from a management studies BA). This minimal information is also sometimes found in science courses, which may simply give a module title such as 'Mathematics for Chemistry'. There are clearly issues here for those who give advice and support to students applying for universities.

The relationship between the mathematical requirements for entry to a course and the mathematics encountered on the course was reviewed in some detail, and the next section of this report consists of a number of subject case studies in which examples of the variability of course requirements and of the mathematics on courses are discussed.

4.4 Case studies of mathematics within higher education courses

There are significant differences between the mathematical prerequisite requirements made by different universities for the same subjects, as shown in Chart 4 below:

Chart 4
Mathematics entry requirements for different subjects in different universities



As can be seen from this chart, the mathematical attainment required for entry to university for a given subject can vary widely. The need for mathematics to be included as an entry requirement might be expected to be related to the mathematics that will be encountered on the course. However, our research reveals that a high level of mathematics is sometimes used as an entry requirement in order to act as a filter to make it easier to ensure that entrants have a high level of general ability. This may apply in 'selecting' institutions. However, the opposite may apply in 'recruiting' institutions, where mathematical requirements for entry may be downgraded from what would ideally be needed given the mathematical demands of the course.

The internet-based survey of university subjects attempted to explore the relationship between mathematical requirements for entry and the mathematics on the course. In some (but not all) cases where information was lacking on the internet, it was possible to make direct contact with university personnel who could give additional information. The following case studies for each of the subjects listed in Chart 4 have been compiled based on this research.

4.4.1 Physics (5 courses)

Requirements in mathematics	A-level Grade A	A-level Grade B	GCSE A	GCSE B	GCSE C	None
No. courses	2	3	0	0	0	0

All the physics courses in our survey require A-level mathematics, and there is substantial mathematical content in all of them – not only in modules labelled as 'Mathematics', but also in modules such as 'Quantum Physics and Relativity' and 'Fundamentals of Quantum Mechanics'.

Our sample of Physics courses did not include any from two of the university groups – the Alliance and Million+ – because none of the six universities in our sample from these groups run a physics course. Indeed, out of the 40 physics courses listed on the *Guardian* website[13], only one is from a post-1992 university.

Mathematics modules on one physics degrees course were summarized as follows:

- Mathematics I (year 1): calculus, complex numbers, vectors, linear algebra and statistics.
- Mathematics II (year 2): vector calculus, scalar and vector fields, electric, magnetic and gravitational fields, partial differential equations, Fourier series and transforms.
- Mathematics III (year 2): differential equations (ordinary and partial) and real and complex matrices.
- Mathematical modelling (option year 1).

Another university also included the following topics:

- Year 2 option mathematics: introduction to group theory; more advanced matrix theory; Cartesian tensors; more advanced theory of differential equations (including solution in power series and expansions in characteristic functions); Fourier transforms; calculus of variations; functions of a complex variable; calculus of residues.

It is clear to those applying that physics departments require mathematics at A-level, and the courses themselves go beyond this. Nevertheless, those teaching on physics degrees have reported

[13] See http://www.guardian.co.uk/education/table/2010/jun/04/university-guide-physics, accessed 20 April 2011

issues to us about the adequacy of the mathematical preparation of their students. As one tutor put it: 'General problem-solving skills are weak – [there is] not so much opportunity for students to practise this at school. This is an area of concern.'

4.4.2 Chemistry (5 courses)

Requirements in mathematics	A-level	GCSE A	GCSE B	GCSE C	None
No. courses	0	0	1	4	0

One course required a Grade B in GCSE mathematics and the rest required Grade C. One of the courses merely stipulated a level 2 Key Skill in Application of Number. There was little information available on the internet about the mathematical content of modules within chemistry degrees, although all except one course included modules in mathematics for chemistry. One course included two modules called 'Skills for Chemists', which focused on physics and mathematics. This course also listed a programme for those without A-level mathematics, which covered numbers and algebra, managing units, the form and use of functions and equations, and an introduction to basic calculus. In terms 2 and 3 all students covered limits, differential calculus, integral calculus and differential equations, followed by work on power series, complex numbers and vectors.

The views of tutors from two of the universities were obtained, and both stated that although the course did not require A-level mathematics, it should do so. As one of them said: 'I would prefer students to have A-level mathematics as some of the higher-level modules have significantly more mathematics than those in 1st year, and we find that students who have A-level mathematics are much better at handling the 1st year work.'

In relation to the difficulties that students have with mathematics, the following were cited:

- Rearranging equations.
- Carrying out simple differentiation and integration.
- Changing between units.
- Plotting graphs.
- Recognizing trends in data.
- Understanding and calculating confidence limits.
- Use of logarithms and exponentials.
- General problem solving.
- 'Guesstimating' order of magnitudes.
- Aligning the 'mathematics' with 'reality'.
- Seeing that the variables in science are the same as the 'x and y' in mathematics.

Most of these topics are in A-level rather than GCSE mathematics.

There are clearly significant mathematical demands within chemistry degrees, and chemistry departments address this by aiming to ensure that their students study mathematics within their course. However, if their mathematics is weak, students will struggle.

4.4.3 Economics (5 courses)

Requirements in mathematics	A-level	GCSE A	GCSE B	GCSE C	None
No. courses	1	1	1	2	0

The mathematics requirements for entry in economics vary from Grade C at GCSE to Grade A at A-level. Two universities require GCSE Grade C, one requires B and one requires A. One university requires A-level Grade A.

GCSE Grade C: The two courses that require a GCSE Grade C have modules on quantitative methods and one lists the 'ability to use quantitative skills in the identification and analysis of economic problems' as a desired outcome of the course; but no detail of content is given.

GCSE Grade A: The university that requires mathematics Grade A at GCSE also states that they encourage applicants to take at least AS mathematics, and if econometrics is studied as a combined degree this university then requires a B Grade at A-level mathematics. In addition, this university states that 'the mathematical and statistical material taught in all other [economics] degrees is designed to be accessible to those with A Grade in GCSE mathematics or its equivalent'.

The mathematical modules listed for this course are:

- Year 1: Introductory Statistics; Introduction to Statistical Theory 1; Mathematics 1 (for econometrics only); Mathematical Techniques in Economics.
- Year 2: Introduction to Statistical Theory 2 (econometrics only); Mathematics 2 (econometrics only); Linear Algebra (option, core for econometrics); Mathematics for Economists (option); Econometrics (option).
- Year 3 options: Mathematical Economics; Time Series 1; Econometric Methods for Research (core for econometrics).

No further detail on the content of these modules is given.

A-level Grade A: The course that requires Grade A at A-level lists a wide range of mathematical topics, as well as statistics. Examples of topics from each of the three years of the course are:

- Year 1: differentiation of ln x and exp(x) product, quotient and chain rules for differentiation; integration of x?1; type I and type II errors; sampling distributions.
- Year 2: linear algebra; regression (simple properties of ordinary least squares estimators; sampling distributions of regression coefficients and correlation coefficients; analysis of and tests for misspecification, specifically serial correlation, heteroscedasticity and structural change).

- Year 3 options: Markov chains; Bayes' Theorem; regression (estimation; matrix formulation of general linear model; Brownian motion; stochastic calculus).

Economics is a subject where there is clearly some diversity in the mathematics encountered on the degree course and, consequentially, the mathematics needed to join the course. One economics professor told us that 'Economics is mathematical modelling'; but this may not be the case in all economics degrees. Another expanded on this by drawing attention to methodological questions within economics, and the danger that areas of investigation are driven by whether a model is mathematically tractable rather than whether it is valid. Mathematics is thus fundamentally important in that those studying in this area need to be capable and confident at using mathematics, but in addition they need an appreciation of the limits of mathematization.

4.4.4 Accounting (6 courses)

Requirements in mathematics	A-level Grade A	GCSE A	GCSE B	GCSE C	None
No. courses	2	0	2	1	1

Information was received on six courses from five universities. One of the courses has no entry requirement for mathematics; two require A-level mathematics at Grade A; of the remainder, two require GCSE mathematics at Grade B and one requires GCSE mathematics at Grade C.

No requirements: The course that does not require any prerequisite mathematics includes the following topics within its 'Financial Mathematics' module:

- Compound growth, discounting, APR, evaluation of capital projects.
- Index numbers (price relatives, weighted indices, RPI).
- Planning production levels (linear programming).
- Project planning (network analysis).
- Elementary probability (e.g. rules of probability, expected value, decision trees, introduction to hypothesis testing).
- Collecting, presenting, analysing and interpreting data (including sampling methods, graphs, charts, basic statistical measures) to produce management information.
- Investigating relationships between variables (correlation, regression using two variables with an introduction to multiple regression).
- Forecasting (time series analysis).
- Key quantitative techniques.
- Use of basic arithmetic (ratios, percentages, fractions, algebra and statistics in management decision-making).

GCSE Grade C: There is little detailed information from the course requiring GCSE Grade C, but one of the modules is titled 'Business Economics, Mathematics and Statistics', and among the list of skills that students are expected to develop is the ability to apply a range of numeracy skills, including an appreciation of statistical concepts, at an appropriate level.

GCSE Grade B: The two sources that require B at GCSE include differentiation among the topics (but not integration), as well as a substantial amounts of statistics. One course also mentions operations research techniques.

A-level Mathematics: The two courses that require A-level mathematics include calculus; one specifically mentions integration.

4.4.5 Computer science and computing (5 courses)

Requirements in mathematics	A-level Grade A	GCSE A	GCSE B	GCSE C	None
No. courses	1	0	1	2	1

One course requires an A Grade in A-level Mathematics, while a GCSE only is required for three of the courses: Grade B and Grade C in two. One of the courses has no mathematics pre-requisite.

GCSE: Two of the courses provide very little detail of the mathematics used. In one of these it may be inferred (and was confirmed by interview with a member of staff) that mathematics is included in the modules 'Foundations of Computing', 'Foundations of Mathematics' and 'Artificial Intelligence'. In the other course discrete mathematics is mentioned. Another course includes 'the mathematical foundations of computer science'; as well as vectors, matrices and statistical methods.

No prerequisite: A third course, for which there is no prerequisite mathematics qualification required, mentions numerical techniques including computation in different number systems, compression algorithms and statistics.

A-level Grade A: The course that requires A-level mathematics specifies discrete mathematics, symmetry groups, linear algebra, Boolean algebra, Fourier transforms, mathematical proof and computational complexity. Mathematics occurs in all three years of this course; in the first year, within modules for 'Computer Architecture', 'Discrete Mathematics', 'Theory 1' and 'Theory 2'; in the second year, in 'Mathematics and Statistics' and 'Logic and Database Theory'; and, in the third year, in 'Computational Complexity' and 'Advanced Mathematical Methods' (as an option).

4.4.6 Biosciences (5 courses)

Requirements in mathematics	A-level	GCSE A	GCSE B	GCSE C	None
No. courses	0	0	0	4	1

None of the sampled courses requires more mathematics than a Grade C at GCSE, though two courses mention mathematics as one of several acceptable A-level sciences. One course does not indicate any mathematics requirement.

One of the courses does not list any modules with an explicit mathematical content. Another has the following modules: 'Mathematical Statistics', 'Statistics' and 'Further Statistics'; but it does not give any details. The other three courses all focus on statistics, with modules titled 'Data Handling and Presentation', 'Quantitative Methods – Natural Sciences', 'IT and Numeracy' and 'Data Handling Skills'. The course that includes 'Data Handling Skills' has two modules described as follows:

1. Designs experiments, performs calculations and manipulates data.
2. How to use statistical techniques, analyse data, use scientific software, design experiments and perform calculations.

In the course for which most detail on mathematics is available, the first year 'Data Handing and Presentation' course seems to be restricted to descriptive statistics, and in the 'Quantitative Methods' course in the second year hypothesis testing and inferential statistics are encountered.

In biochemistry there is a need to appreciate the importance of mathematical models. As one professor of biochemistry explained to us, an important part of the conceptual development within the subject is to move from a model of the chemical situation to a mathematical model; and it is this move to the need to think mathematically that is difficult for those students who have not studied much mathematics.

4.4.7 Psychology (7 courses)

Requirements in mathematics	A-level	GCSE A	GCSE B	GCSE C	None
No. courses	0	0	0	5	2

Five of the sampled courses require GCSE Grade C in mathematics, although one course strongly encourages mathematics at A-level. Courses all include modules on research methods or quantitative methods. There is an emphasis on selecting the most appropriate statistical test from among a range of types. Two courses include training in SPSS, a statistics software package.

One lecturer felt that an important skill was the metamathematical skill of knowing when to apply a mathematical technique.

4.4.8 Criminology (6 courses)

Requirements in mathematics	A-level	GCSE A	GCSE B	GCSE C	None
No. courses	0	0	0	5	1

Apart from one course that does not specify any mathematics, all courses require Grade C mathematics at GCSE.

Three courses include a module called 'Research Methods', one on 'Quantitative Methods', one on 'Using Social Research Data' and one mentions 'The Scientific Study of Crime', although it is unclear how much mathematics is studied in this module. Two courses specifically mention statistical inference, confidence intervals and hypothesis testing.

The content of the 'Quantitative Methods' course is given as:

- Measures of central tendency and dispersion.
- Elementary probability.
- Common distributions.
- Sampling distributions.
- Elementary statistical inference.
- Estimation and hypothesis testing.
- Contingency tables, bivariate correlation and regression.
- Understand and describe the content of statistical tables derived from published statistical sources.

4.4.9 Sociology (5 courses)

Requirements in mathematics	A-level	GCSE A	GCSE B	GCSE C	None
No. courses	0	0	0	0	5

None of the sociology courses investigated specifies any level of prior mathematical attainment.

All courses include modules in research methods or data analysis. Three of the courses give some detail of the content of these modules, including statements like 'simple statistical and graphical techniques', with an emphasis on sampling. One course also includes training in the use of SPSS for handling and analysis of survey data. There is no evidence of the study of different types of statistical test; the focus appears to be on the design of surveys and the presentation and summary of the data arising from them.

4.5 Mathematical competency in higher education

During the last 20 years, concern has been expressed about the inadequacy of the mathematical skills possessed by undergraduates. Assuming this observation to be accurate, there are two possible explanations.

- The demand for new undergraduates with mathematical skills has outstripped the supply. This is undoubtedly the case. The supply of 125,000 students with level 3 mathematics is much the same as it was 30 years ago. However, in that time there has been a massive expansion in higher education, and many subjects have become more mathematical. The mismatch between supply and demand makes it inevitable that many university lecturers are critical of the mathematical competence of their new undergraduates.
- There are those who claim that standards have fallen. It is the case that there have been reductions in content of both GCSE and A-level mathematics, and it is also the case that more high grades are now awarded. Consequently, it is hard to argue that the standard represented by a particular grade in one of these qualifications has not fallen; that, however, is not the same thing as the standard of mathematics attained by particular students. On this we have no evidence.

There are two distinct targets of this concern: those who have studied mathematics to A-level and those who have not.

A major report that gathered together evidence on this issue was published by the Engineering Council in 2000 as *Measuring the Mathematics Problem*, a collection of papers and discussions from a seminar held at Cambridge University in 1999. This report was about those who had followed a course in A-level mathematics. It described the perception at that time of a decline in mathematical competence of HE students, and also outlined some of the strategies used to deal with this, most notably the establishment of support programmes.

Some teachers of chemistry degree courses expressed concerns about the ability of even those with good grades at A-level mathematics to use mathematics in their subject area. However, this needs to be balanced by the more major concern that about one third of those embarking on such degrees have not taken A-level mathematics at all.

Recent research by the STEM Advisory Group (Onion 2011) has described the steady increase in numbers taking A-level mathematics, from 45,000 in 2002 to 70,000 in 2010. The research reports on the proportions of those following various degree subjects who passed A-level mathematics, and there have been significant increases in these proportions in most subjects.

The second target of concern is those who join university courses having studied no mathematics beyond GCSE. Even with the welcome rise in the number of those taking A-level mathematics, it is still the case that nearly 40 per cent of chemistry students, over 50 per cent of architecture students and over 60 per cent of computer science students have not studied A-level mathematics. Given the patterns of education in this country, it is inevitable that a very large percentage of those without A-level will not have studied any mathematics after 16.

This is of particular concern for Initial Teacher Education (ITE) degrees. Of those enrolling on primary ITE courses – and these will of course be the mathematics teachers of children in primary schools – only 10 per cent have passed A-level mathematics. In connection with this, Professor Dylan Wiliam reports that when he analysed data from the Second International Mathematics Study (SIMS), he found that the achievement of fourth-graders in different countries was highly correlated with the proportion of students who continued with mathematics past the age of 16[14]. William's explanation for this was that since the primary school teachers teaching fourth-graders (9-10 years old) had themselves studied mathematics beyond 16, they were better able to teach the content.[15]

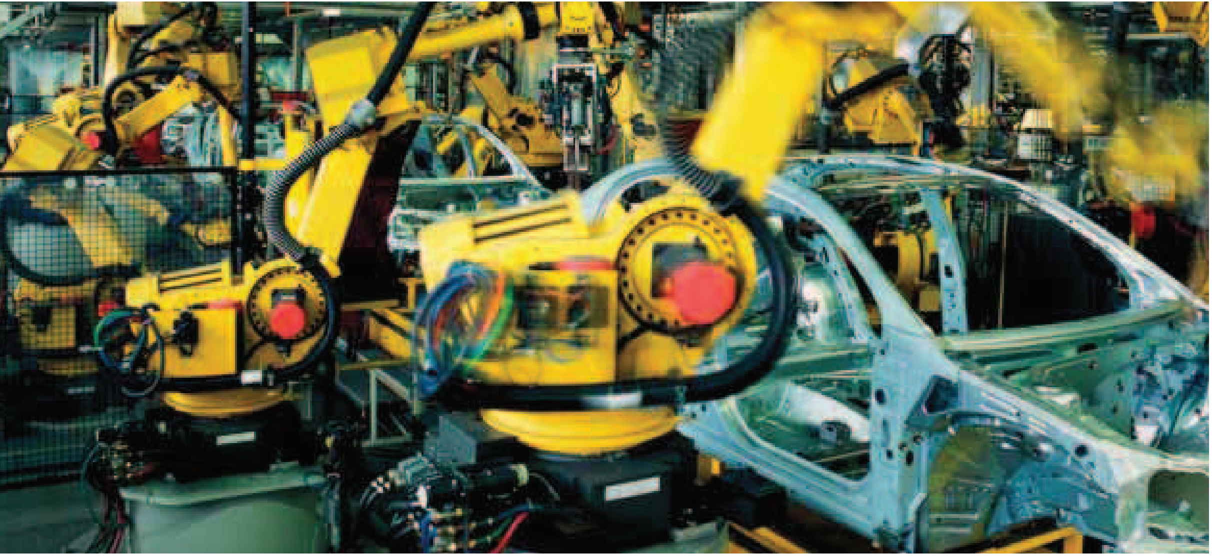
Many tutors of HE courses that have significant mathematics content and mainly recruit those with GCSE mathematics are concerned about the mathematical preparation of their students and the unreliability of GCSE Grades as a predictor of mathematical competence. This was summarized by one member of a biology department as follows:

> *Many students have also had little opportunity to apply and practice their mathematical skills between GCSE and Stage 1, leading to a lack of proficiency and confidence in applying skills they were previously able to use. This was borne out by the results of diagnostic testing carried out in this department some years ago. Results for a diagnostic test based on GCSE intermediate level mathematics indicated that at GCSE an A* Grade was the only indicator of a good score on the diagnostic test. At all other grades (A–C) a range of test scores from fail to good was obtained. In contrast a poor, or even unclassified mark, at AS or A2 level was a better indicator of a good score. This indicates that many of the problems we see in applying numerical skills may result from a lack of confidence and practice.* (Foster 2008)

This perception was reinforced by a chemistry lecturer's response to our questions: 'Those who haven't done maths since GCSE always use the excuse "But I haven't done that for nearly 3 years and I have forgotten how to do it."'

There is a significant amount of compensatory activity in universities to deal with the mathematical problems that students have, through voluntary attendance at mathematics workshops. Universities are coping with this by setting up systems to provide mathematical support to their students. The Sigma model developed by Coventry and Loughborough is a well-established example. There are many *ad hoc* solutions in which departments deliver their own 'maths for the subject' courses. For example, use is occasionally made of successful school teachers to deliver these courses.

14 Private communication from Professor Wiliam, Emeritus Professor of Educational Assessment, Institute of Education.
15 A list of the companies visited and the industry sector they belong to is given in Appendix 2.



5. Mathematics in the workplace

5.1 Methodology

The employment sector is very diverse, and it was necessary to find ways of achieving a representative set of views across the wide range of interests. The employment sector can be stratified in a number of ways, each of which has a bearing on the type of mathematics likely to be required:

1. Sector. The type of activity undertaken by an industry will clearly impact on the role of mathematics within it. Government statistics are collected in relation to six large groups: primary sector and utilities; construction; manufacturing; non-marketed services; distribution and transport; business and other. Several of these cover a very wide range of skills requirements.

2. Type of job. The level of mathematics required of an individual will also relate to the demands of the particular job role within the industry. Jobs are classified into one of nine categories: managers; professionals; associate professionals; personal services; sales occupations; administrative and clerical; skilled trades; transport and machine operations; and elementary occupations.

3. Size of organization. This often has a bearing on the kinds of job required. Large organizations are more likely to need relatively small numbers of highly technically qualified staff. Small organizations may have staff who have to undertake a very wide range of functions. It is often said that 97 per cent of employers have fewer than 50 employees, and while this is true it is also the case that the 3 per cent of companies that employ more than 50 staff employ 72 per cent of the workforce (Humphries 2004).

As the starting point it was decided to carry out interviews with employers and employees from about 25 companies. Most were approached using the good offices of sector skills councils, whose brief concerns training and workplace development. This was supplemented by using other organizations or direct contacts with companies.[16]

The range of mathematics involved was deliberately chosen to be as wide as possible. We have reviewed research on numeracy and have interviewed staff whose involvement with mathematics in the workplace does not go beyond basic numeracy, as well as staff who are graduates in mathematics.

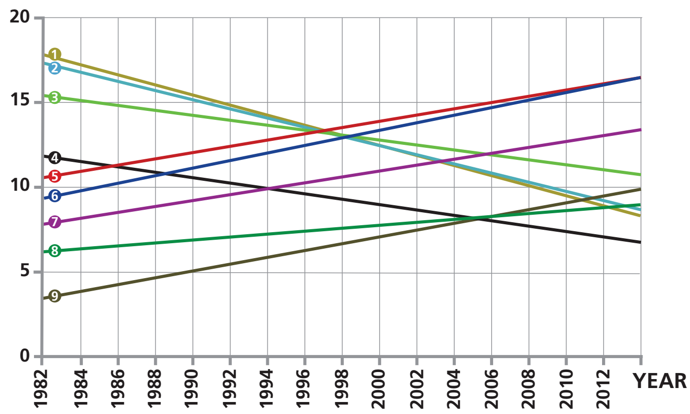
5.2 Changing patterns of employment

The balance of jobs within the global workplace has changed in the last thirty years and is continuing to do so. Table 3 and its accompanying Chart 5 indicate the changes in different occupational groups in the UK between 1982 and projected 2012 (Humphries 2004).

Table 3:
Numbers of workers in each occupational category based on data for 1982 and projections for 2012

Fewer jobs	1982	2012
1. Elementary occupations	4.5m	2.8m
2. Skilled trades	4.4m	2.8m
3. Administrative and clerical	3.9m	3.4m
4. Transport and machines	3.0m	2.2m
More jobs	**1982**	**2012**
5. Managers	2.7m	4.9m
6. Professionals	2.0m	4.0m
7. Associate professionals	2.4m	4.9m
8. Sales occupations	1.6m	2.7m
9. Personal services	0.9m	2.9m

Chart 5: (see Table 3 above)
Patterns of occupational change based on data for 1982 and projections for 2012[17]



16 A list of the companies visited and the industry sector they belong to is given in Appendix 2.
17 Numbering of lines corresponds to category numbers in Table 3. The vertical axis expresses the data as a percentage of the total workforce.

Table 3 and Chart 5 show two trends:

1. The number of people in medium-to high-skilled jobs is increasing.
2. The number of people in unskilled or semi-skilled employment is decreasing.

It is important to note that categories 5, 6 and 7 will soon account for 13.8m jobs, whereas in 1982 they only accounted for 7.1m.[18] It is safe to assume that the vast majority of the jobs in these categories will require qualifications beyond level 3, thus providing a major driver for the expansion of higher education. It is not an automatic implication that this increase in higher-level jobs will also mean that a larger number of people will need to be qualified to a higher level in mathematics, but it is reasonable to expect that this may be so. It is therefore necessary for a greater number of individuals to be able to cope with increased mathematical demands in the workplace.

In relation to the mathematics needed at work, we were impressed by the number of respondents who said that the mathematics they actually used at work was of a lower level than that to which they had studied, but that studying at this higher level was not wasted as it gave them confidence and security in applying the mathematics that they needed to use. If this is generally the case, then a challenge for the application of mathematics in the workplace may be to ensure that, in their learning prior to work, individuals achieve a level of mathematics at least one level above that which they are likely to encounter in their job role.

In addition, most employers mentioned their concern at the inability of many employees to *apply* the mathematics that they knew. For some, this is seen as a consequence of an excessive emphasis in schools on 'teaching to the examination', which they believe has increased in recent years.[19]

However, the use of mathematics in the workplace does not just concern high levels of mathematics, but also basic levels of mathematics. Large-scale Skills for Life surveys indicate that a substantial proportion of the adult population lack basic skills in numeracy. As the Leitch Review commented in 2006:

> *Surveys, like the Skills for Life Survey conducted in 2003, assess people's basic skill levels using a variety of literacy and numeracy problems corresponding to five levels.*[20] *In 2003, 21 per cent (6.8 million) of the working age population in England lacked Entry Level 3 numeracy skills. (Stationery Office 2006)*

Research into employers' perceptions of the skills possessed by employees reinforces this conclusion. In 2010, the UK Commission for Employment and Skills (UKCES) reported on the extent to which employers are dissatisfied with the skills of the applicants for work (skills shortages), and also noted the skills gaps among their own employees[21]. In addition, the Confederation of British Industry (CBI) recently published a report outlining the needs for more mathematics skills in industry and asking for all young people to continue with numeracy and mathematics in post-16 education and training.

From the perspective of our concern with *mathematics* in the workplace, it is important to note that surveys like this do not ask employers to identify gaps or shortages in 'mathematics'; and it is of course possible that poor mathematical skills may contribute to the perceived gaps in 'technical and practical skills' and in 'problem-solving', but it is not possible to judge the extent to which this is true on the basis of these figures.

5.3 Case studies of mathematics in the workplace

As a result of the visits and telephone interviews undertaken with 25 employers/companies, a number of case studies on the use of mathematics in the workplace have been developed. As the interviews progressed, various recurrent themes emerged, and these have formed the basis of the summary of the case studies below. The detail of the case studies is included in Chapter 6 of this report.

These themes are clearly sufficiently important to be incorporated into the design of a future provision and curriculum for mathematics.

5.3.1 Mathematical modelling

The development and application of mathematical models occurs across a range of industries. It was common to find individuals who used a model that was developed elsewhere in the company or to a software package that was essentially a mathematical model. In the first case study below, our respondent used models developed by other people and had also devised his own model. The other case studies show a wide variety of complexity in terms of the mathematical modelling involved. In general, there are relatively few staff who develop models (e.g. actuaries in an insurance company) and many more who use them (e.g. insurance sales staff). The case studies were:

- Engineering and regulatory requirements of a water company.
- Mathematical modelling developed by a graduate trainee in a bank.
- Modelling carried out by a station manager.
- Modelling the cost of a sandwich.
- Monitoring the performance of the production of paper in a paper mill.

18 It is of course a moot point whether these projections are still valid following the recent recession, but the general point of a significant increase since 1982 in 'level 3+' jobs remains true.

19 This was most clearly expressed by the national training manager (of apprentices and new entrants) for a major transport company who said that 'some have not been educated, but instructed to pass an exam. This is especially evident in the second year, when they need to apply mathematics to practical problems'.

20 The UK uses five levels to measure literacy and numeracy skills: Entry Levels 1, 2 and 3, and Levels 1 and 2. The Moser Report (Moser 1999) identified Entry Level 3 numeracy as the standard necessary to function at work and society in general. An example of an Entry Level 3 numeracy skill is being able to add or subtract money using decimal notation, or being able to work with fractions. It is important to note that the research for this ACME report would indicate that modern employment demands a significantly higher level of mathematical competency.

21 A 'skills gap' implies an area where individuals within the existing workforce have lower skill levels than are necessary to meet business or industry objectives, or where new entrants lack some of the skills required for them to perform effectively. A 'skills shortage' is where there is a lack of adequately skilled individuals in the labour market.

5.3.2 Use of software packages and coping with problems

Software packages are now very widely used in industry. The availability of computers has not just changed *how* work is done, it has changed *what* work is done. This change requires staff to be more mathematically competent. Although employees may not be required to undertake routine calculations, they are required to undertake higher cognitive tasks such as interpreting the meaning of the computer-generated results of calculations. The first case study below is an example of a situation in which manual calculations had to be made because of a breakdown of the IT equipment. There are several examples of this kind of occurrence. More common, however, is the need to be able to input the correct data and make sense of the output, as is done in the second case study. One quality manager pointed out that the computer does all the calculations, but without the computer it would simply not be economical to carry out such a large number of the calculations (which then require interpretation and action). A chartered engineer made a very similar point, and this is described in the third case study. We also came across several individuals who had developed their own spreadsheets to deal with particular jobs. The case studies were:

- Oil extraction company: response factors on a gas chromatograph.
- Limestone quarry: proportions of impurities.
- Using IT packages in the water industry to model a number of different drainage scenarios: the use of dispersion models of sewage.

5.3.3 Costing (including allocating costs and managing disputes)

It has always been important to ensure that costs are apportioned correctly. With the increase in outsourcing of functions that are regarded as 'non-core', and with contracts that contain penalty clauses for non-compliance, the identification and correct allocation of costs has become even more important. The four case studies here show the importance of correct calculation in allocating responsibility for costs appropriately. In three of the cases something did not go according to plan – a pump appeared not to be working properly; a train was late; the foundations of a bridge were shifting. In each case it was necessary to carry out a detailed calculation to determine the cost of putting the situation right, and its appropriate allocation. It is necessary for those involved in this work to understand what the calculation is about, even if much of the work will be carried out using a spreadsheet. In the fourth case, a standard procedure using a spreadsheet was used to calculate whether the company was liable for penalties for not meeting the specification. The case studies were:

- Piling for the building of a bridge.
- Contract cleaning in a hospital.
- Heating and ventilating consulting engineer.
- Duty manager of a major rail terminus.

5.3.4 Performance indicators and the use of ratios

Performance management, through the use of appropriate performance indicators, is increasingly common, and it is important that those who devise the indicators understand what behaviours they are intending to affect. There are two examples from the insurance industry. In addition, indices are used in a variety of scientific contexts. The example given here is from the work of a dietician who, as she said, 'now sees the need for the mathematics I learnt'. The case studies were:

- Developing an index that compares insurers.
- Ratios in an insurance company.
- Glycaemic index.

5.3.5 Risk

The need to deal with risk arises in many industries, and the four examples listed below show the breadth of the range of examples. Risk ranking is used to identify and manage the risks involved in complex multi-stage processes. The accurate estimation of the risks that apply to members of different collectives is at the heart of the insurance industry. In experimental work, the regulatory requirements are based on the management of risk, and risk is a vital issue in clinical governance in the NHS, where a manager expressed his concern that many users of the data are not statistically literate. The case studies were:

- Risk ranking in the offshore oil industry.
- Calculating risk in insurance.
- Risk assessment in experimental work.
- Clinical governance.

5.3.6 Quality control and statistical process control

The widespread use of statistical process control (SPC) in industry means that increasing numbers of staff need to be able to read and act upon charts that show how product measures vary. In one instance the production manager claimed that there was 'no mathematics' in his factory; yet SPC charts are very widely used there. In this case the procedure had become so much part of the daily routine of the workplace that it was no longer perceived as mathematics. The case studies were:

- Furniture manufacturer.
- Measuring the deviation of railway lines.
- Developing logarithmic multi-variable regression at a paper mill.
- Machine downtime at a metal enclosure manufacturer.
- Measuring flow in the oil industry.

6. Workplace Case Studies

In the previous section of this report we referred to case studies arising from our interviews with employers. In this section we describe these in more detail. They are based on interviews with employers and employees from 25 organizations from many industry sectors. The organizations range in size from the NHS (allegedly one of the three biggest employers in the world) to a small catering company. Inevitably, case studies from such a diverse group of companies vary in length and detail.

The range of mathematics involved was deliberately chosen to be as wide as possible. In the interviews with employers a number of issues emerged that were common across several employers and sectors, and these have formed the basis of the classification of case studies that we have used. Examples of the use of specific mathematical topics in different industries are given in Appendix 3.

6.1 Mathematical modelling

The development and application of mathematical models is very common and occurs across a range of industries. It was common to find individuals who used a model that was developed elsewhere in the company, or a software package that was essentially a mathematical model. In the first case study below, our respondent used models developed by other people and had also devised his own model. The other case studies show a wide variety in complexity of modelling.

6.1.1 Case study: Engineering and regulatory requirements of a water company

A major requirement of the director for engineering and regulation (DER) for a water company is to work with computer-based mathematical models.

For fixing prices, there is a mathematical model developed by the regulator of the industry; it is accessed through a software package called Reservoir, which enables water companies to calculate the appropriate price increase in relation to retail prices index (RPI) inflation. A parameter denoted by k, which determines how much above or below inflation prices can be set, is calculated by this software. The mathematics underpinning this is very complex and is based on microeconomic data.

A different package is used for the development of capital spending. This is called the Capex Incentive Scheme (CIS) and it affects what the regulator thinks a company should spend on capital projects.

Finally, there is the model of water supply and demand balance, which the director developed himself and which is essentially an algebraic formula. The following parameters are included in the model:

- Metered and unmetered customers (an assumption is that metered customers use 10% less than unmetered ones).
- Occupancy rates of households.
- Per capita consumption.
- New properties.
- Void properties.
- Leakage.

This model predicts the overall demand expected. The actual amount used will vary by about ±5 per cent, depending on the weather. It is necessary to be able to cope with peak demand, which is (empirically) 1.4 x average demand. The DER had to develop this model himself and his knowledge of the relevant mathematics was absolutely crucial.

6.1.2 Case study: Mathematical modelling developed by a graduate trainee in a bank

This mathematics graduate has been working in several different areas of the bank as part of her traineeship. She feels that her mathematics degree was not essential to get the job but has helped on several business management projects. She has developed the following mathematical models:

1. Modelling costs of sending out bank statements versus going online.
2. Modelling price and volume of mortgages to estimate the key ratio of loan to value as house prices change (if house prices go down then the ratio goes down).
3. Clearance of cheques. There is a long-term downward trend in cheque use, which was also affected by a large drop in the Christmas period; hence there is a need to see if there is a permanent effect because of this.

4. Modelling the use of Link ATM machines – this is a joint project with KPMG.

5. Competitor analysis. One critical project was to correctly identify like for like costs when Lloyds and HBOS merged. This was made more difficult because HBOS had recategorized some clients from business to retail.

6.1.3 Case study: Modelling carried out by a station manager

The duty manager for a major rail terminus developed a spreadsheet for the 'tanking' of trains (ie providing enough water for lavatories, etc.). This model has to deal with situations where tanking is needed for the return journey as well as for the outward journey.

6.1.4 Case study: Modelling the cost of a sandwich

The food operations controller of a catering company that supplies sandwiches and lunches both through mobile vans and as special orders for external customers has developed a spreadsheet that enables the cost of sandwiches and similar items to be calculated. It was necessary as part of this work to estimate the cost of onions in hamburgers, which was done by finding out how many burgers can be filled from one onion. The most difficult parameter to estimate for the model is the cost of labour.

6.1.5 Case study: Production of paper in a paper mill

A spreadsheet is used to monitor performance of the production of paper, and this is based on a model developed by the quality manager. Sometimes the system fails and things break down. This requires a reassessment of the model that is used – it may be that the system has changed and therefore the assumptions are no longer valid, requiring a shift in the regression equation.

6.2 Use of software packages and coping with computer/software malfunction

Software packages are very widely used in industry. The first case study below is an example of a situation when manual calculations had to be made because of a breakdown in the IT equipment. There are several examples of this kind of occurrence. More common, however, is the need to be able to input the correct data and make sense of the output, as is done in the second case study. One quality manager pointed out that the computer does all the calculations, but without the computer it would simply not be economical to carry out such a large number of the calculations – which then require interpretation, and then action. A chartered engineer made a very similar point, and this is described in the third case study.

6.2.1 Case study: Oil extraction company

When software developed a fault it was necessary to carry out calculations manually (e.g. calculate the response factors on a gas chromatograph by drawing the graph and then calculating the area under the curve).

6.2.2 Case study: Limestone quarry

To make up a load of limestone, rock is blasted from different locations (known as 'shots'), and this is analysed in the laboratory to indicate the proportions of impurities that are present. The impurities are lead, magnesium oxide and sulphur, all of which must be below a certain concentration. For example, lead must not exceed 10 ppm in the aggregate leaving the site; and results for shots have been known (exceptionally) to be as high as 1200 ppm. In order to get an acceptable product, material from different shots is blended. The proportions from each shot are calculated by a spreadsheet which was devised by the quality manager of this site.

6.2.3 Case study: Using IT packages in the water industry

IT has significantly changed the way in which engineers work in the water industry. One specific example: to get the right answers to questions about drainage they have to calculate the amount of rain falling and the way in which the rain is then able to be passed on through the drainage system. Before computers this was a hand calculation that took days; you can now do it in seconds. Because the calculation was so long-winded, people used to go for a very conservative position so as not to have to calculate it again. Now it is possible to model a number of different scenarios and explore different strategies for dealing with rainfall.

A second example of the application of IT and mathematics is in the use of dispersion models of sewage. For these they use charts for the hydraulic designs of channels and pipes, which are in the standard reference book. This uses the Colebrook–White equation, which is based on the Prandtl–Kármán theory of turbulence. A key factor here is *Ks* – the roughness coefficient of the material of the pipe. There are many empirical formulae that have to be used and there are two people who look at the work. One looks at the validity of the calculation; the reviewer then checks the validity of the model. With IT it is possible to do a lot of modelling and therefore to have a much finer approach to safety. This makes it all the more important that the assumptions and input values are correct.

An interesting sidelight on the way in which very similar mathematics is found in widely different situations is provided by this example and the case study cited in Section 6.3.3. The graphical representation of flow of sewage through pipes in relation to the roughness coefficient is exactly the same in form as that for the flow of air in ventilation systems, which is used by heating and ventilating engineers. In both cases a series grid of intersecting straight lines plotted on a log–log graph enables the engineer to read off the appropriate diameter of pipe to be used for a given set of circumstances (CIBSE 2009).

6.3 Costing (including allocating responsibility and managing disputes)

The four case studies here show the importance of correct calculation in allocating responsibility for costs appropriately. In three of the cases something did not go according to plan, and in each case it was necessary to carry out a detailed calculation to identify the responsibility for allocating the cost of putting the situation right. It is necessary for those involved in this work to understand what the calculation is about, even if much of the work will be carried out using a spreadsheet.

6.3.1 Case study: Piling for the building of a bridge

As part of the construction of a new bridge a number of piles had to be sunk into the land on either side of the river. Because the land was marshy the piles had moved. It was very important that the construction engineer did his calculations correctly to demonstrate that the blame for the problem did not lie with his company.

6.3.2 Case study: Contract cleaning in a hospital

A major facilities management company has won the contract for cleaning one of the London teaching hospitals. Housekeeping staff regularly check the cleanliness of the areas for which they are responsible. There are 100 domestic items to be graded, and each one is given a pass or fail. A member of staff known as the 'help desk manager' compiles spreadsheets of the data on four different factors. The combined score of these factors, appropriately weighted, determines the payment that the company receives from the hospital. The help desk manager is not required to have a mathematics qualification but does make effective use of the spreadsheet. In a recent month, the company lost £35,000 in payment because of low scores. The weightings of different factors in the spreadsheet are determined by high-level negotiations between the company and the hospital.

6.3.3 Case study: Heating and ventilating consulting engineer

Mathematics is important to the consulting engineer, because often he can be in the middle of a dispute about the performance of equipment that he has recommended to the client. This occurred in relation to a pump that appeared not to be performing to the specifications outlined by the consulting engineer, who carefully recalculated all the figures to demonstrate that their specification was correct. The problem was resolved when it emerged that the operator was not using the pump to its full capacity.

6.3.4 Case study: Duty manager of a major rail terminus

A major part of the work of the duty manager at the station involves the 'delay budget'. If a train is delayed, ATOC (the Association of Train Operating Companies) fines the company responsible. The correct allocation of responsibility for a delay is thus a fundamental requirement. Delay can be caused by:

- Station staff – this is within the duty manager's budget.
- Train crew – this is within the train crew budget.
- Delay on line – if a signal this is within the Network Rail budget, or if by a delayed train it is within another company's budget.
- Train failure – this is within the engineering budget.

ATOC use a program called TRUST, which indicates all the delays on the network. The duty manager reviews TRUST reports to make sure that any penalties have been fairly allocated. He passes on information to the 'delay attribution people' at First Great Western (FGW) head office. This impacts on the company's bill to ATOC. He also has responsibility for the overtime budget and the sickness budget, and is also audited on safety and security.

6.4 Performance indicators and the use of ratios

Performance management through the use of appropriate performance indicators is increasingly common and it is important that those who devise them understand what behaviours they are intending to affect. The two examples below both come from the insurance industry.

6.4.1 Case study: Developing an index that compares insurers

The global carrier division has the task of estimating the security of 'carriers' (ie the insurance companies with which an insurance broking company deals). The company has developed its own quality index. This is prepared every year based on returns from 7000 company associates across the world. The index is built up of qualitative and quantitative aspects. The work involves understanding basic statistics, including range and measures of average and of dispersion.

6.4.2 Case study: Ratios in an insurance company

The following ratios are important for the accountancy section of an insurance broker:

- Profit/assets.
- Interest/debt.
- Earnings/dividend.
- Income/head count.
- This year's growth/last year's growth.
- Expenses/income.

The accountant's job is not merely to calculate these ratios but to use them to 'tell a story'.

6.4.3 Case study: Assigning glycaemic index values

A nutritionist in a hospital explained the importance of mathematics to her. Her BSc in nutrition and dietetics involved chemistry, biochemistry, mathematics and statistics; as she said, 'I'm now learning why we did the maths'. For her degree thesis she assigned glycaemic index (GI) numbers to foods. This is a number assigned to a food that represents the rate at which a food affects someone's blood sugar – the highest GI is for glucose, which is 100.

There are formulae on which people's nutritional requirements are calculated. These involve biometric factors such as age, weight and gender, and other factors related to stress, such as if patients are in intensive care.

Interestingly, this nutritionist was educated in Ireland, where students feel obliged to study at least some mathematics in their pre-university course.

6.5 Risk

Uncertainty is inherent in all sectors. It is necessary to develop methods of allowing for uncertainty by trying to estimate in some way the risks involved in an enterprise and the consequences of unexpected outcomes. The first three examples below show the variety of different circumstances in which risk is calculated, while the fourth reflects concerns in the health sector that risk is not always well understood.

6.5.1 Case study: Risk ranking in the offshore oil industry

Risk ranking takes place for each specific activity (e.g. tank entry) using a ranking matrix ship/shore compatibility in loading cargo at appropriate load rates within a set period of time. There are many variables to be considered and, although not necessarily mathematical in nature, they need the ability to parallel-process significant amounts of technical data.

6.5.2 Case study: Calculating risk in insurance

Key calculations include the risk factor, which is the loss rate – the ratio of pay-out on claims to income from premiums. There is substantial use of Excel formulae in this work, although the re-insurance officer would also carry out mental calculations at an early stage.

6.5.3 Case study: Risk assessment in experimental work

In a company that undertakes laboratory assessments of new products, the risk assessment of experiments is the responsibility of the head of regulatory affairs. He works to ensure that everything proceeds according to the appropriate regulations. There are two major regulatory documents – REACH (Regulation for Registration, Evaluation, Authorisation and Restriction of Chemicals) and AGRI (Chemistry regulations).

Substantial mathematics is involved, including physical and chemical indicators. These include toxicity end points, octanol water partition coefficients, water solubility, half-life of soil degradation, absorption and desorption coefficients, and mobility. These feed into mathematical models of how materials end up and the risk that is involved. The company does not create these models – they are agreed by the regulatory authorities – but its staff need to understand and use them. Models are used at different levels. For example:

1. Operator exposure calculations.
2. Budgeting and accountancy.
3. To satisfy scientific and regulatory requirements.

The models involve modelling waste, human exposure, operator exposure, bystander exposure and eco-tox. By way of example, it was explained that Richard' equation is used to describe water flow through the soil. This is a chromatographic flow through different media to produce PEC (predicted environmental concentrations). Some models are drift models, which are relatively simple, but flow models are more complicated. The key parameter is what is known as the degradation half-life, which is an indicator of the extent of the adsorption to the soil of material.

6.5.4 Case study: Clinical governance within the health service

Risk calculations are also a major aspect of clinical governance within the health sector, and there are concerns about the capabilities of those who have management roles in this area. A senior manager within a major teaching hospital expressed his concerns as follows: 'Generally, people don't understand statistical variation.'

At a very straightforward level, there is an issue of poor communication when people use a table rather than a graph to illustrate data. The example given was of DNAs ('did not attends') for patients receiving treatment for different cancers at different locations. The impact of the figures would have been far greater had they been graphed instead of merely tabulated. However, there are also many examples of ineffective graphical displays seemingly put into a report simply to add variety. The example given was of complaints data, as part of a clinical governance and risk management report.

There are pockets of understanding, such as patient statistics data teams, but many users of the data are not statistically literate. The problem is that there are many professionals within the NHS whose training and background has not given them a sophisticated grasp of the uses that statistics can be put to. Many people are involved in compiling reports through bodies like PICANET (Paediatric Intensive Care Network) and tasks include the coding, collection, display and interpretation of data.

The drive for public accountability is increasing the need for people who can handle data properly. It also means that the public needs to be better educated.

6.6 Quality control and statistical process control

The widespread use of statistical process control (SPC) in industry means that increasing numbers of staff need to be able to read and act upon charts that show how product measures vary. In one instance the production manager claimed that there is 'no mathematics' in his factory; yet SPC charts are very widely used. In this case the procedure has become so much part of the daily routine of the workplace that it is no longer perceived as mathematics.[22]

6.6.1 Case study: Furniture manufacturer

A new door was introduced with a thickness of 18 mm. New hinges that required hinge holes of 12 mm were introduced, which

[22] This observation is supported by the work of Hoyles, Noss et al (2010) on Technomathematical Literacies. It has also been recognised by the 'embedding project' which was commissioned by Skills for Logistics and carried out by Alpha Plus. This project identified ways in which numeracy and literacy skills were embedded within National Occupational Standards for the logistics industry and has developed 'ghost' units in numeracy.

left only a very small margin for error at the rebate end of the door. Because of this, the doors were being broken by the hinge at the fitting stage. The solution was to replace the hinges with specially imported 10 mm hinges, but this still required careful quality control and tight tolerances, which was where SPC came in. The success of SPC in dealing with this problem led to it being introduced for all the company's processes. It has led to a significant reduction in faults at the fitting stage.

The furniture manufacturer produces SPC charts for a range of parameters, including hinge drilling control (depth and measurement from door edge), dimensions of door (length, width and squareness), beam saw (length, width and squareness) and chest rails (depth of holes and drill position from top edge).

6.6.2 Case study: Measuring the deviation of railway lines

On-track machines (OTM) are used to monitor the on-track geometry by travelling quickly along the tracks. Ten different measurements are taken. Measurements are taken every 6 inches, and the standard deviation of measurements for every 35 meters of track is calculated. The equipment produces a range of displays, which are like a SPC chart. It is interesting to note the strange mix of measures of length used here.

The OTM delivery manager has developed spreadsheets that display the information in terms of categories of quality of track: good; satisfactory; poor; very poor; red alert. Action is based on this. Each stretch of track has a number of inspections per year based on the speed with which trains travel and on the gross tonnage carried. The engineer needs to understand geometry, as well as statistics, in order to deal with the results.

6.6.3 Case study: Developing logarithmic multi-variable regression at a paper mill

An important part of the quality control work is to develop logarithmic multi-variable regression to predict a property of the paper from one variable to another (e.g. thickness, which indicates strength). The detailed calculations are done by spreadsheet but the quality manager has to understand how it works.

One example is the use of statistical calculations to set up quality parameters C_p and C_{pk}, which are measures of spread and accuracy, respectively (as described in the work of Hoyles et al. 2002). The computer program works it out and produces the graphs. The quality manager examines the output, on which he or she will decide what to do.

6.6.4 Case study: Machine downtime at a metal enclosure manufacturer

Workers in the tool room use SPC with quality parameters C_p and C_{pk} as part of the continuous improvement process. The work involves constant learning as the tool room is always developing new solutions.

The production department makes substantial use of SPC charts. A key measure is machine downtime (technical availability of machine is the measure used and 83% is a typical normal value, representing 17% downtime). Some SPC charts are input manually while others are automatic, and when the system is moving out of control a traffic light system operates (amber and red).

6.6.5 Case study: Measuring flow in the oil industry

This involves, for example, forecasting of product flow in pipelines from offshore. The engineer looks for variability and trends in machine performance (against 'performance curves') and then generates a maintenance activity to correct it. Actual pump performance is measured and referenced against the manufacturer's curves. If there is a discrepancy then all variables are considered in terms of how they impact on that performance.

6.7 Non-routine work

Several respondents emphasized the need to be able to deal with non-routine applications of mathematics (for example in construction). In manufacturing, maintenance staff often have to do non-routine calculations, which require confidence in the use of mathematics.

6.7.1 Case study: A 'fudge' at a metal enclosure manufacturer

This example is of an apprentice being entrusted with a challenging non-routine project. One example he gave was when they did a 'fudge' to enable a process to happen without the correct materials, which led to a log-jam of material, which then required a proper solution to be prepared.

The apprentice also described a current piece of work that involves the torque setting on compressed air tools. The previous engineer, who has now retired, used a setting that was far too high. Two apprentices are now experimenting with the compressed air tool to find the appropriate settings for different circumstances. Design and quality are also involved, and it 'has all been quite a hassle'. They are measuring and recording data so that settings will be correct in the future. The underpinning theory of the tool involves relations between volume of air passing through in a given time, diameter of pipe and pressure.

7. Conclusions

This research gave us the opportunity to talk to a large number of people from higher education and the world of employment. In this final section of the report, we summarize and compare the outcomes of the research, highlighting the common features and the differences.

SUPPLY AND DEMAND

MATHEMATICS IN HIGHER EDUCATION

- Data on HE acceptances suggest that some 180,000 of those accepted will encounter a significant amount of mathematics on their courses. An additional 150,000 students in the social sciences will also encounter some mathematics on their courses. The total demand figure for mathematically competent students is thus over 300,000 per year, but the supply is only 125,000.

MATHEMATICS IN THE WORKPLACE

- Surveys of employers, such as the CBI report 'Making it all add up' (2010) also point to a similar deficit at the more elementary levels of mathematics.

THE PROVISION OF MATHEMATICS POST-16

MATHEMATICS IN HIGHER EDUCATION

- The recent increase in the numbers of those studying A-levels in mathematics and further mathematics is to be welcomed, but this in itself will not solve higher education's need for more mathematically competent students. For many students aspiring to HE, a variety of mathematics courses beyond GCSE is needed. Recent initiatives such as A-level Use of Mathematics and the Evaluating Mathematics Pathways project explore this idea and can provide valuable insight.

MATHEMATICS IN THE WORKPLACE

- Information on numbers studying mathematics on vocational courses post-16 is inadequate, but our research indicates that relatively few of those following vocational courses study mathematics. This needs to be remedied.

THE MATHEMATICS ACTUALLY REQUIRED ON THE JOB

MATHEMATICS IN HIGHER EDUCATION

- The mathematics required for recruitment to courses is sometimes less than what would ideally be required, and is a pragmatic response to ensuring student numbers are sufficient.
- The different mathematics backgrounds required (and in many cases, the lack of a mathematics requirement) for the same subject sometimes lead one to suspect that, say, criminology at University A must be very different from criminology at University B.
- Many courses in higher education require statistics at a level well beyond GCSE.

MATHEMATICS IN THE WORKPLACE

- Several interviewees felt the need to have studied to a level higher than that habitually used at work in order to be able to apply the mathematics effectively. One respondent said: 'I don't use much of the maths from my degree – but the maths degree I did is incredibly valuable in helping me develop new approaches to the work.' The mathematical techniques actually used in the work may be relatively low-level, but the situations in which they are applied are often conceptually challenging.
- Serious concerns were expressed about the lack of statistical sophistication of many decision makers in the health sector.

MATHEMATICS IN HIGHER EDUCATION

MATHEMATICS IN THE WORKPLACE

THE MATHEMATICS NEEDED FOR PROGRESSION

Mathematics in higher education

- There are many courses, with entry requirements limited to Grade C at GCSE, which make significant mathematical demands on their students at some stage during the course.

Mathematics in the workplace

- Progression at work may depend on understanding concepts that rely on a knowledge of mathematics that is not directly used in the workplace. For example, the mathematics underpinning the installation of ventilation systems and the theory needed for polymer technicians are quite challenging for those who work in these areas.

MATHEMATICAL SKILLS AND QUALITIES NEEDED

Mathematics in higher education

- Several lecturers complained that students have been drilled to pass their school examinations and are unable to use their mathematics in the new contexts they are meeting at university.
- For many subjects 'metamathematical' skills are important (ie knowing when to use a technique as well as how).
- Mathematical modelling is an important part of many subjects; for instance, in psychology, it can be used to interpret and evaluate mathematical models of cognitive processes, and derive empirical hypotheses from models. For one professor, 'economics is mathematical modelling.'

Mathematics in the workplace

- It is the application of mathematics skills that employers most value, and several times we heard the comment that entrants to the workforce have been drilled in passing examinations rather than in understanding mathematics and its application.
- Several respondents emphasized the need to be able to deal with non-routine applications of mathematics. Examples were given from engineering, construction, transport, laboratory work and finance, where individuals have needed to draw on their own mathematical skills to solve a problem.
- The use of modelling is widespread. This may involve working with and interpreting an existing model, or, at a more senior level, creating one's own model.

THE USE OF ICT

Mathematics in higher education

- Much of the data collected in subjects such as psychology is processed on software packages such as SPSS. This makes it all the more important that students understand the basic statistics that are being applied, including the assumptions underlying any tests the computer is carrying out and how to interpret the results.

Mathematics in the workplace

- The use of computers is universal in the workplace, and, as a direct consequence, the demand for mathematical skills has increased. Computers allow situations and processes to be monitored more closely, but the data and information created have to be interpreted. This often involves understanding an underlying mathematical model.

FURTHER SKILLS FOR PEOPLE TO BE EFFECTIVE USERS OF MATHEMATICS

Mathematics in higher education

- Within the social sciences there is concern about the marginalization of quantitative methods and the lack of understanding of the role of mathematically based arguments within social sciences disciplines.

Mathematics in the workplace

- The importance of communication skills was spelled out by an accountant in an insurance company, who said: 'A chartered accountant sees his job as understanding and telling a story through the numbers.'
- In relation to the context within which mathematics is used, one senior manager in the IT industry said: 'In addition to maths, we look for problem-solving, working with others and communication. It is important that these qualities are addressed within mathematics degrees – through giving students opportunities to work in teams, to make presentations and so on. There is a danger that the (misnamed) "soft skills" are underdeveloped within STEM subjects.' This could be extended to the way mathematics is taught in school.

MATHEMATICS IN HIGHER EDUCATION

ATROPHY OF KNOWLEDGE

- Many HE students have experienced a two-year gap from their last encounter with mathematics. In the case of those enrolling onto a PGCE course this may be as long as five years.

PERCEPTIONS OF MATHEMATICS

- A graphic illustration of the role of mathematics within other subjects and the challenges university teachers face was given to us by a professor of biochemistry, who explained: 'The modelling process has three phases: a pictorial representation, a chemical equation and the mathematical model. Understanding how the different stages in the pictorial model feed in to give the ultimate mathematical formulation – typically a hyperbolic equation – is often difficult for students. They need fluency in moving between the visual representation of a situation and its symbolic description – and I don't know how to teach students this. But in our experience university mathematicians are not the right people to help on this either.'

ADVICE AND GUIDANCE

- The information that is readily available from universities on the level of mathematics encountered within degree courses is often minimal (although there are exceptions to this) and does not go beyond the titles of units. This has major implications for the careers advice that students receive when they are making subject choices at 16.
- One of our HE respondents told us: 'Schools enrol people for foundation maths GCSE as a safety device to ensure they get a Grade C, thus ensuring a good score in the performance tables. Many may be capable of performing at a higher level. The decision on level is taken at the end of year 9. It is clear that the combination of the two tiers of maths with the high stakes of the league tables is to incentivise schools to encourage students not to attempt higher-level GCSE maths – as an insurance policy.' The teachers involved need to understand the profound effect this decision may have on the career opportunities that will subsequently be open to those students.

MATHEMATICS IN THE WORKPLACE

ATROPHY OF KNOWLEDGE

- There is widespread concern among employers about the ability of recruits (at all levels) to apply the mathematics that they should know.

PERCEPTIONS OF MATHEMATICS

- Some respondents did not perceive some of the techniques that they used (such as statistical process control) as 'mathematics'. In a sense, the mathematics used by many individuals in industry is 'hidden'. One of our respondents used the term 'practical logic' to describe the (mathematical) thinking he used.
- There were several cases of people saying that the mathematics is mainly done by computer software, but they need to make manual calculations when the computer breaks down. Examples of this have been found in the finance and petroleum distribution industries.

ADVICE AND GUIDANCE

- The importance of mathematics in apprenticeship programmes may not be evident in the information available to careers specialists. More use could be made of the experience of apprentices, as in the example of an apprentice that we interviewed who went back to his old school to give a talk on mathematics to 15-year-old pupils on an engineering day.

References

BIBLIOGRAPHY

Asset Skills (2008a) *Making the Business Case for Numeracy in the Workplace*, updated by NCTEM, www.ncetm.org.uk/numeracy

Asset Skills (2008b) *Maths for Work*, Summary of Literature Review, Asset Skills

CBI (2010), *Making it All Add Up: Business Priorities for Numeracy and Maths;* CBI

CBI (2010). *Ready to Grow: Business Priorities for Education and Skills;* Education and skills survey 2010; CBI

CIBSE (2009) *CIBSE Guide B: Ventilation, Air Conditioning and Acoustics,* Chartered Institution of Building Services Engineers, London

Coben, D. (2003) *Adult Numeracy: Review of Research and Related Literature,* National Research and Development Centre for Adult Literacy and Numeracy, Institute of Education, London, November, p. 10

Construction Skills (2007) *Construction Skills Learning Exercise: Practice Questions.* Construction Skills. August

CPSA (2006) *Technical Guide,* Concrete Pipeline Systems Association

Croft, A. C, Harrison, M. C, Robinson, C. L. (2009) 'Recruitment and retention of students – an integrated and holistic vision of mathematics support', *International Journal of Mathematics Education in Science and technology,* vol. 40, no.1 pp. 109–125

DBIS (2010) *Science For Careers: Report of the Science and Society Expert Group*, Executive Summary, Report of the Science and Learning Expert Group, Science and Mathematics Secondary Education for the 21st Century, DBIS, February

DCSF (2008) *Mathematics Performance in Primary Schools: Getting the Best Results*, Report By The Comptroller And Auditor General, HC 1151, Session 2007-2008, Department for Children, Schools and Families, London, 19 November

DfES (2003) *The Skills for Life Survey: A National Needs and Impact Survey of Literacy, Numeracy and ICT Skills,* DfES Research Brief RB 490, DfES, London

Engineering Council (2000) *Measuring the Mathematics Problem,* Engineering Council, London

Foster, K. (2008) 'Biomathtutor evaluation: a tutor's perspective', *Bioscience Education,* vol. 11, June, www.bioscience.heacademy .ac.uk/journal/vol11/beej-11-r5.aspx, retrieved 27 March 2011

HMSO (1982) *Mathematics Counts,* Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr W. H. Cockcroft, HMSO, London

Hodgen, J., Pepper, D., Sturman, L. And Ruddock, G. (2010) *Is the UK an Outlier? An International Comparison of Upper Secondary Mathematics Education,* Nuffield Foundation, London

Hoyles, C. Wolf, A., Molyneux-Hodgson, S. and Kent, P. (2002) *Mathematical Skills in the Workplace Final Report to the Science, Technology and Mathematics Council.* Institute of Education, University of London, London

Hoyles, C., Noss, R. and Kent, P. (2010) *Improving Mathematics at Work: The Need for Techno-Mathematical Literacies,* Routledge, Abingdon

Hudson, C. (2007). *Maths4Life: Working Mathematics for the 21st Century? A Discussion Paper,* published by the National Research and Development Centre for Adult Literacy and Numeracy, www.ncetm.org.uk/resources/8570, October

Humphries, C. (2004). *Skills in a Global Economy,* City & Guilds

Jones, H. (2009) 'Fundamental problems', *THES*, 2 July

Learning and Skills Council (2007) *National Employers Skills Survey (NESS),* LSC

LSIS (2009) *Numeracy in Train to Gain: A Guide for Train to Gain Brokers and Providers,* Learning and Skills Improvement Service

MacInnes, J. (2009) *Proposals to Support and Improve the Teaching of Quantitative Research Methods at Undergraduate Level in the UK,* ESRC, www.esrc.ac.uk/_images/Undergraduate_quantitative_research_methods_tcm8-2722.pdf, retrieved 28 March 2011

MEI (2009) *A Level Mathematics and Further Mathematics: An Investigation into the Reasons for Increased Uptake in 2009,* Mathematics in Education and Industry, www.mei.org.uk/files/pdf/A_Level_uptake.pdf, retrieved 28 March 2011

Moser. C. (1999) *Improving Literacy and Numeracy: A Fresh Start,* Report of the Working Group Chaired by Sir Claus Moser on Behalf of the Department for Education and Skills, London: HMSO, London, www.lifelonglearning.co.uk/mosergroup

Onion, A. J. (2011) A-level mathematics and progression to university: A contribution from the STEM Advisory Forum; *Mathematics Today,* vol. 47, no. 1, February, pp. 24–25

Parsons, S. and Bynner, J. (2005) *Does Numeracy Matter More?* National Research and Development Centre for Adult Literacy and Numeracy, Institute of Education, London

Pell, G. and Croft, T. (2008). 'Mathematics support – support for all?', in *Teaching Mathematics and its Applications,* Oxford University Press, Oxford

Polya, G. (1957) *How to Solve It,* Anchor Books, New York, NY

QCA (2007) 'Introduction to mathematics', in *Functional Skills Standards,* QCA

QCA (2009) *AS/A Level Subject Criteria for Mathematics,* QCA Consultation draft, for first teaching from September 2012, QCA, April

Rees, R. M. (1973) *Mathematics in Further Education: Difficulties Experienced by Craft and Technician Students,* Brunel Further Education Monographs

References

Royal Society (2011) *Preparing for the transfer from school and college science and mathematics education to UK STEM higher education. A 'state of the nation' report.* London, Royal Society.

SCORE (2008) *SCORE Report GCSE Science: 2008 Examinations,* SCORE Education

SCORE (2009) *Progression Routes in Science, Technology, Engineering and Mathematics (STEM),* Final Report for the Science Council, SCORE Education, September

Semta (2009) *SDDP Line of Learning Statement for the Diploma in Science,* Semta, Watford

Skemp, R. (1976) *Relational Understanding and Instrumental Understanding 1,* Department of Education, University of Warwick, Warwick

Smith, D. (2010) *Endpoint: Mathematical Skills Required,* Education in Chemistry, May

Steedman, H. (2005) *Apprenticeship in Europe: 'Fading' or Flourishing?* CEP Discussion Paper 710, Centre for Economic Performance, London School of Economics and Political Science, London

Tariq, V. (2003) *Diagnosis of Mathematical Skills among Bioscience Entrants,* Diagnostic Testing for Mathematics, LTSN Maths TEAM Project, Queen's University, Belfast

Tariq, V. (2004) 'Numeracy, mathematical literacy and the life sciences', *MSOR Connections,* vol. 4, no. 2, pp. 25–29, May, Queen's University, Belfast

Tariq, V. (2004) 'Sum things don't add up: Tackling the numerical skills deficit', *The Bioscience First Year: Putting Research into Practice,* Centre for Bioscience event, available at www.bioscience.heacademy.ac.uk/ftp/events/brighton/tariq.pdf, September

Stationery Office (2004) *Making Mathematics Count,* report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education,The Stationery Office, London, February

Stationery Office (2006) *Prosperity for All in the Global Economy – World Class Skills:* Final Report of the Leitch Review of Skills, The Stationery Office, London, December

UK Commission for Employment and Skills, Belt, V., Campbell, M. and Giles, L. (2010) *Skills for Jobs: Today and Tomorrow, volume 2: The Evidence Report.*

Vidal Rodeiro, C.L. (2007) *A level subject choice in England – patterns of uptake and factors affecting subject preferences* Cambridge Assessment, UK Commission for Employment and Skills

Wake, G. (2008) *Keeping Open the Door to Mathematically Demanding Programmes in Further and Higher Education,* Teaching and Learning research briefing, Manchester University, Manchester

Wake, G. (2010) *Mathematics Learning, Identity and Educational Practice: The Transition into Higher Education,* Teaching and Learning research briefing, Manchester University, Manchester

Watts, A. (2005) C*ounting on Numbers – Squaring the Numeracy Divide?* Proceedings of the Science Learning and Teaching Conference, University of Warwick, Warwick

Williams, P. (2008) *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools,* Final Report, June

Wolf, Alison (2011) *Review of Vocational Education: The Wolf Report*, London, DfE

Appendices

APPENDIX 1 ACKNOWLEDGEMENTS

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The Mathematical Needs Advisory Group

- Dr Keith Ison, NHS
- Helen Joyce, The Economist
- Professor Duncan Lawson, Joint Mathematical Council
- Professor John MacInnes, University of Edinburgh
- Professor Richard Noss, Institute of Education
- Professor David Spiegelhalter FRS, University of Cambridge
- Richard Wainer, CBI

Contributors

- Dipankar Basak of Black and Veatch Consulting Engineers
- Mike Bialyj of Construction Skills
- David Braben of Frontier
- Ken Butcher of CIBSE
- Janet Carter of the University of Kent
- Nick Clarke of Tarmac
- Diana Coben of King's College London
- Diane Cochrane of Wolverhampton University
- Katy Cole of Medirest, King's College Hospital, London
- Alan Courts of Rittal
- Tony Croft of Loughborough University
- Dave Curwen of the RAF
- Gina Donaldson of Canterbury Christ Church University
- Pat Drake of University of Sussex
- Lesley Earle and colleagues of Huntingdon Life Sciences
- Catherine Elliott of Summit Skills
- Dick Evans of City & Guilds
- Terry Fennell of the Association of Learning Providers
- Sue Foster of Royal Bank of Scotland
- James Fothergill of the CBI
- David Fox of Pfizer
- Robert Freedman and colleagues of the University of Warwick
- Paul Freeman of Smurfit Kappa
- Neil Gordon of the University of Hull
- Luke Graham of the TDA
- Michael Grove of More Maths Grads
- Paul Hambley of LLUK
- Graham Hasting-Evans of UKCES
- Sue Hill, Chief Scientific Officer of the National Health Service
- Jeremy Hodgen of King's College London
- Kevin Howat of BAM Nuttall
- Celia Hoyles of the Institute of Education, University of London
- Alastair Hudson of Financial Services Sector Skills Council
- David Hudson of Process Skills
- Steve Hunt of Steve Hunt Ltd
- John Hutton of Friends Provident
- Keith Ison, Head of Medical Physics and colleagues of St.Thomas's Hospital, London
- Lisa Jarvis of Lantra
- Sue Johnstone of First Great Western
- Sir Michael Latham, past Chairman, CITB Construction Skills
- Hugh Lawlor, Chair of the Science Diploma
- Duncan Lawson of Coventry University
- David Lloyd of Sharp's, Wolverhampton
- Andrew Mackness of Norma's Catering
- Johanna Macnamara Willbourne of Network Rail
- Peter McKee, Chief Executive, TRL Technology
- Paul McKelvie of UKCES
- Lucy Milton of the National Skills Academy for the process Industries
- Andrew Naylor of Henry Colbeck
- Maureen O'Mara of Skills for Logistics
- Mathew Philip of London Metropolitan University
- Tony Pringle of Cogent
- Jon Purdy of the University of Hull
- Kieran Quill of Cogent
- Paul Robinson of Conoco Philips
- Daniel Sandford Smith, Gatsby Foundation
- Chris Sangwin of the University of Birmingham
- Gary Shoesmith of Willis Insurance
- Michael Sich of City & Guilds
- Rosemarie Simpson of Skills for Health
- Sharon Simpson of Asset Skills
- Howard Sissons of Wates
- Lester Sonden of Sutton and East Surrey Water
- Bill Sutton, Brian Thornton and John Harris of SEMTA
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- Judith Swift of the TUC
- Richard Thomas of the RAF
- Simon Thompson of the University of Kent
- Anthony Tomei of the Nuffield Foundation
- Sir Glenn Torpy, former Chief of the Air Staff
- Stephen Uden of Microsoft
- Dr Jean Venables, Past President, Institution of Civil Engineering
- Geoff Wake of the University of Manchester
- St.John Walker of Skillset
- Tim Watts of Tim Watts Consulting
- Vicky Williams of Compass
- Anna Wyatt of Lantra
- Jiang Zhiang of Bournemouth University

APPENDIX 2 CONTACTS AND INTERVIEWS WITH COMPANIES

Sector	Company	Contact	Region	Interview
Primary	Water company	Engineering and regulatory director	Southeast	Interview with director & HR manager (*n* = 2)
	Off-shore oil company	Training manager	Northeast	Interview with training manager & 4 staff (*n* = 5)
	Quarrying company	National training manager	West Midlands	Interview with training manager & 2 staff (*n* = 3)
	Consulting engineers	Principal engineer	Southeast	Discussion with 6 engineers (*n* = 6)
Manufacturing	Pharmaceutical manufacturer	Research manager	Southeast	Interview with research manager (*n* = 1)
	Paper mill	Managing director	West Midlands	Interview with MD & 3 staff (*n* = 4)
	Metal processing company	Managing director	Southwest	Interview with MD & 5 staff (*n* = 6)
	Animal research company	Head of scientific training	East of England	Interview with HST & 12 staff (*n* = 13)
	Furniture manufacturer	Managing director	West Midlands	Interview with MD & 1 staff (*n* = 2)
Construction	Consulting engineers	Managing director	Northwest	Interview with MD & 2 staff (*n* = 3)
	Construction company	Local manager	Southeast	Interview with manager & 3 staff (*n* = 4)
	Construction company	Professional development manager	Northeast	Telephone interview (*n* = 1)
Business and other	Insurance brokers	Technical training director	East of England	Interview with TTD & 3 staff (*n* = 4)
	International IT company	Head of skills and economic affairs	Southeast	Interview (*n* = 1)
	Bank	Human resources manager	Scotland	Interview with HR manager & 7 staff (*n* = 8)
	Insurance company	National sales learning and development manager	Southeast	Interview with manager, plus sixteen questionnaire returns (*n* = 17)
	Computer games company	Managing director	East of England	Telephone interview (*n* = 1)
	Contract catering company	Head of human resources	Southeast	Telephone interview with follow-up on-site interviews with 6 staff at a hospital (*n* = 7)
	Small catering company	Managing director	Southeast	Interview with MD & 3 staff (*n* = 4)
Non-marketed services	RAF	Training manager and head of apprenticeships	West Midlands	Interview with training manager plus 4 staff. Telephone interview with head of apprenticeships (*n* = 6)
	Teaching hospital	Senior medical physicist	Southeast	Interview with SMP & 4 other staff (*n* = 5)
	NHS HQ	Chief scientific officer	Southeast	Interview (*n* = 1)
Distribution	Transport company	Managing director	Northeast	Interview with MD & 1 staff (*n* = 2)
	Rail operator	Human resources manager	Southeast	Interview with HRM & 4 staff (*n* = 5)
	Network Rail London	National training manager	Southeast	Interview with NTM & 2 staff (*n* = 3)

APPENDIX 3 SPECIFIC MATHEMATICAL TOPICS IN DIFFERENT INDUSTRIES

A3.1 Computer games

For the managing director of a computer games company the absolutely essential elements of mathematics for his work start with matrices and computational techniques. It is essential that students have a basic familiarity with mathematical algorithms and with computational algorithms:

> *Not being able to understand matrices in this business is like a car mechanic who doesn't understand engines – ie useless.*

This interview was followed up by discussions with staff working in three universities delivering courses in computing that are closely linked with the games industry, and the following mathematics topics were identified as essential:

- Basic Vector algebra.
- Matrix algebra.
- Linear algebra.
- Transformations.
- Linear spaces.
- Numerical methods.
- Algorithm design.

Those without A-level mathematics do a Mathematics module in the first year. Its content includes:

- Algebra.
- Trigonometry.
- 2D equations.
- Logic.
- Differential equations.
- Tangents to lines and curves.
- Introduction to matrices.

The focus is very much on applications (e.g. union of sets in setting up an Access database).

A3.2 Construction

The professional development manager of a major civil engineering company said:

> *The mathematics we want of graduates is trigonometry, algebra, numeracy, and some statistics and matrices. They need to understand graphs. But no calculus. They need to be very good at the mathematics they do and they need to apply it in a practical situation.*
>
> *The more difficult stuff is done on bought-in computer software and then they have to interpret and apply the results. Different levels of mathematics are undertaken by different companies within the industry supply chain. Thus, consulting engineers develop proposals – which require the highest level of mathematics, whereas companies that have an operational role must implement the proposals and need to understand the mathematics needed to do this.*

The site manager for a different civil engineering company pointed out that:

> *A lot of modifying and calculating takes place when estimating and tendering for work. There are standard parameters, such as the amount of bricks that one bricklayer can lay in a day, but these are always being modified because of site-specific factors. Another example is the calculation of how long it would take to raise a steel building based on the number of items to be put together.*

His colleagues gave several examples of the need to re-calculate on-site. From a site engineer:

1. *Re-aligning what had been put in the wrong place. Everything had to be recalculated.*
2. *Piling for the building of a new bridge. Because the land was marshy the piles had moved. It was very important that the site engineer did his calculations correctly to demonstrate that the blame for the problem did not lie with his company.*
3. *Piling for a new library building. In this case the building required geothermal heating, the pipes for which had to be incorporated among the piles. The chalk environment upset the initial calculations, requiring extra piles in order to meet the heat capacity of the building.*

From a mechanical and electrical services manager:

> *Lifting a generator with a mobile crane. The circuits on the crane tripped because it was overloaded. There is a high safety margin. They had to recalculate whether it would be safe to proceed using a less sensitive trip-out. It was, and they were able to do the job.*

A3.3 Finance

A great deal of use is made of 'elementary mathematics' applied in situations of great complexity. The graduate trainees are usually highly qualified in mathematics (A-level, plus a degree that includes a significant amount of mathematics). They say that the advanced mathematics that they have experienced enables them to apply mathematics effectively. A key capability is the ability to develop and use mathematical models.

A3.4 Railway maintenance

The factors that affect tracks include the throw on bogies going round a bend and the centre thrust. The design figures are put into a spreadsheet; parameters are calculated and checked at the end of a job. Static measurement is done by individuals and dynamic measurement by the NMT train. This work involves a good understanding of physics, which requires constant mathematical thinking.

An apprentice engineer described examples of how the wear on rail flanges is caused by wheels and how, following a fatal derailment in Cumbria in 2007, there is a new design to reduce this. He also described how new rails are welded into place so that the length is correct, applying the linear expansion formula. He outlined how he has to communicate results through a hierarchical chain of command on various areas of concern: performing stress restoration; side wear and head loss trends for different sites; different temperatures for stressing rails.

One example where he was required to use his mathematics knowledge was when a new rail was being laid, and the temperature changed suddenly during the day. He had to recalculate the parameters involved and work out a new extension for the rail.

Another important aspect of rail design is that of transitions. All curves on railway lines are arcs of circles, but in order to link the arc to a straight there is a 'transition curve'. There are two kinds of these in use: a cubic parabola and a clothoid spiral – which has the property that it gives a constant rate of angular acceleration. The engineer in charge of maintenance has to calculate the required curve using a spreadsheet and communicate the results to the team installing the rail.

The engineer interviewed felt that a lack of mathematical skills in engineers does limit their ability to carry out their role fully. He felt that there is a lack of interest in developing sound mathematically based processes for recording and showing the results of monitoring (e.g. on a spreadsheet), and consequently the work is not carried out as efficiently as it might be.

A3.5 Mathematics in aerospace engineering

Semta (the Sector Skills Council for Science, Engineering and Mathematical Technologies) is carrying out research into the mathematics used by engineers in the aerospace industry by asking individuals to identify which of a list of topics they use in their work, and how important these topics are in terms of frequency of use and proficiency. The results so far have shown the following (Semta 2009):

- The job roles vary considerably in the overall 'importance of mathematics', from quite low for the engineer (systems designer) to very high for the principal engineer (manufacturing engineering manager). In addition, the roles vary in the importance of each mathematical item, which is an indication of the importance of the mathematics that is required for a role, even if not all areas of mathematics are needed. On this measure the chief engineer (electrical systems) scores highly and the senior engineer (product owner and support) has a low score.
- When the topics are ordered by overall score the four most important areas are: statistical techniques; statistical methods; areas and volumes; and critical path analysis.
- A high score is achieved by a combination of frequency of occurrence of an item and its individual score. Hence, when items are ordered by mean score, which is a measure of how important they are for those who need them, then the highest-scoring ones are mental estimation, basic numeracy and mental arithmetic.
- When the items are ordered by 'level of difficulty', based on whether items are basic numeracy, GCSE level, A-level or above A-level, the majority of items appear to be at A-level.

A3.6 Mathematics within engineering design

A design engineer in one of the factories visited prepared a list of mathematics topics in design, as follows:

- **GENERAL DESIGN**
  - **Component fitment** using relatively basic mathematics to calculate designs to customer specifications.
  - **Parametric relationships** (e.g. calculating spacing for holes).
  - **Plastic snap-ins:** Different equations for developing snap-in components – to calculate the force required (e.g. 50 N) for a given geometry.
- **MECHANICS**
  - **Static –** analysis, stress (tensile, shear), deflection.
  - **Dynamic –** harmonic frequency, stress, deflection, impact forces.
- **TOLERANCE ANALYSIS**
  - **Worst case.**
  - **Root sum squared.**
- **DESIGN ANALYSIS**
  - **Open area percentage:** This indicates how much air flows through the container and requires a spreadsheet to be developed.
  - **Component areas:** Calculating how many racks fit into a shipping container.
  - **Rack removal:** Trigonometry. There are angles to be considered when a ramp is used to load a rack on to a container, in order to achieve the correct clearance.
  - **Component assembly mass:** Employees work with the folded item and the software works out the unfolded dimension. There may be several different specific amendments that may be required; in particular the bend radii need to be pre-set to ensure correct folding. There is a relationship between thickness of steel and bend radius, which is read off a table. Also, the total mass of a component is required by the customer.

A3.7 Mathematics in chemistry

There is a need for those working in chemistry, in various industrial contexts, to use and apply a substantial amount of mathematics. Although much of this is routinized through the use of spreadsheets, there is still a need for those working in the area to understand the mathematics involved, as is illustrated in the following case study.

The animal and plant biology manager of a laboratory testing company has to calculate the appropriate amounts of radiochemicals to order. As these are very expensive he must systematically go through the whole synthesis process and know what the yields are at each stage, and work back from the amount at the end. He then has to cost the components. A typical order is between 1 and 2 curies of radioactivity. In his scientific role he has to measure radioactivity from beta rays within samples, multiplying up, involving weights and volumes and the distribution of radioactivity. He uses scintillation counts to measure radioactivity and do a 'simple sum' to get a sum of what actually happened. In order to calculate the amount of metabolites present in the end, he has to do a chromatographic analysis of the extracts, involving reverse-phase HPLC. This enables him to quantify the different components.

Therefore the job involves, in the first phase, establishing the material balance (the total radioactivity involved); in the second phase, chromatography (to identify the number of specific components and how much of each); and in the third phase, identification of the components using reference standards, if necessary using spectroscopic techniques as well. There is a great deal of spreadsheet work, using a library of validated spreadsheets.

In one of the case study, a review of the mathematics requirements for undergraduate placements was carried out. This provided a very useful list of the mathematics required of people working in chemistry laboratories. The Discover Maths for Chemists website developed by the Royal Society of Chemistry **(http://discovermaths.rsc.org)** is a response to concerns expressed by the industry about the mathematical competence of graduates entering the industry.

The list of mathematical topics necessary for the biosciences placements was as follows, with the additional requirements for physical sciences in italics:

- Good grasp of arithmetic calculations (e.g. moles, molarity, dilution).
- Knowledge of basic statistical concepts (e.g. mean, median, mode, frequency, probability).
- Ability to calculate compound dilution factors and determine necessary volumes to achieve these dilution factors.
- Ability to use/rearrange simple algebraic equations and confident use of brackets.
- Basic understanding/awareness of statistical significance testing, and appreciation of experimental error and uncertainty.
- Ability to plot basic graphs (either using computer programme or by hand).
- Confidence with interconversion of units.
- Confidence with rough estimates and order-of-magnitude calculations.
- Confidence with decimal places and significant figures.
- *Understanding of proportion, fractions, ratios and indices.*
- *Understanding of logarithms and exponentials, and ability to manipulate equations containing these functions.*
- *Knowledge of the principles of integration and differentiation, and their use in deriving key scientific concepts and principles.*
- *Familiarity with the concept of functions and stationary points.*
- *Ability to solve simple simultaneous equations.*
- *Solid knowledge of basic trigonometry and geometry (2D and 3D).*
- *Competence at using equations/statistical tests, understanding underlying assumptions, requirements for validity, and the effects of changing variables within these equations.*
- *Understanding of the principles of combining uncertainties.*
- *Understanding of data distribution (e.g. Gaussian).*
- *Use of spreadsheets for plotting graphs, calculations.*
- *Confidence in using mathematics in a wide range of experimental and theoretical contexts.*

This list was developed by the director of external partnerships for a major pharmaceutical company. In order to do this he invited twelve of his colleagues to identify the mathematics needed in their work. There was enormous disparity here, in that some felt they used a great deal of mathematics while others felt they used hardly any, although all were doing very similar jobs. Many had clearly so absorbed the mathematics that they'd forgotten that they had ever needed it! This echoes the comments made by many at work and by Hoyles et al.'s (2010) research about the invisibility of mathematics.

A3.8 Apprentices in the RAF

The RAF training school at Cosford was visited. The apprentices here are following various programmes in engineering, for all of which mathematics is essential, and normally a GCSE Grade C in mathematics is required for entry. If a candidate does not possess this qualification, satisfactory performance in the RAF's own entrance test may allow them to proceed. The instructors on the course, however, are highly critical of the preparation that GCSE mathematics provides, especially in relation to topics like fractions. As a result, 30 hours of remedial mathematics is also included within the programme.

In addition, the officer responsible for apprentices across the RAF was interviewed. He is responsible for 14 different apprenticeships and supports the view held by the instructors about the inadequacy of preparation from GCSE. This is of concern especially as a Grade C in GCSE mathematics provides exemption from having to study Application of Number, when it is precisely the numerical skills that many apprentices lack. He has hopes that the introduction of Functional Skills will ultimately improve the situation.

The mathematical needs of the different types of apprenticeships followed in the RAF can be summarized as follows:

Engineering, avionics, mechanical technician, ICT technician, photo-imaging:
Significant amounts of algebra, trigonometry and work beyond GCSE.

Information technology, catering, public service, administration, fire fighter:
Some algebra, statistics and fluency with number to a good GCSE level.

Security, warehouse and storage:
Fluency with number, ratios and measures.

It should be noted that pilots need many mathematical skills, including knowledge of algebra, trigonometry and vectors; in addition, they need to have rapid recall and good ability to carry out mental calculations in emergency situations.

A3.9 Mathematics in the National Health Service

For many of those who are attracted to work in the NHS, mathematics is not necessarily one of their strengths. Many have 'scraped through' the mathematics. However, mathematical capability, at several levels, is a vital part of many jobs within the health service, and errors can have fatal consequences. The following are mathematical needs as perceived by a senior member of staff responsible for scientific staff within the health service. Those working in the health service need to understand:

- What numbers mean – for example the difference between 'nano' and 'micro'.
- The importance of variation (indeed, a greater understanding of statistical concepts in general, including linear regression and probability).
- Risk – the move towards increased involvement of patients in their treatments means that both medical staff and the public need a better understanding of the concept of risk.
- Assumptions underpinning mathematical models, which are the source of formulae used in spreadsheet calculations.
- Fractions, percentages and ratios.
- The difference between precision and accuracy.

The National Health Service itself needs to have a better understanding of the range of mathematical needs within its workforce.

APPENDIX 4 GLOSSARY OF TERMS USED

During the course of the research several different terms have been used, sometimes with overlapping meanings, and we list here summary definitions of these terms.

Numeracy

It is clear from the quotation from the Leitch report in Section 5.2 that in its consideration of skills in the workplace the term *numeracy* is used to imply lower levels of mathematics. This use would be contested by many of those working with adult learners who see the term *numeracy* as having a much wider application. In this research we have covered areas that are described both as mathematics and as numeracy.

What is regarded as *mathematics* and what as *numeracy* is partly a question of professional cultures. There are those who would argue each of the following positions: numeracy is part of mathematics; numeracy is separate from mathematics; and mathematics is included within numeracy[23]. For this Mathematical Needs project, references to mathematics and to numeracy are equally relevant. The following definition of *numerate* (rather than *numeracy*) enables the term to be applied to a wide range of contexts, and does not confine numeracy to elementary uses of mathematics:

> *to be numerate means to be competent, confident and comfortable with one's judgements on* ***whether*** *to use mathematics in a particular situation and if so,* ***what*** *mathematics to use,* ***how*** *to do it, what* ***degree of accuracy*** *is appropriate, and what the answer means in relation to the context. (Coben 2003)*

This implicit definition would include a wide range of levels of mathematical complexity within the term numeracy. However, the term numeracy is often used to imply lower levels of mathematics (below GCSE Grade C). When used in the Leitch report (Stationery Office 2006), in UKCES reports, and in LSC and Sector Skills Councils reports, it does usually mean the lower levels of mathematics.

In this report, we have sought, wherever possible, to use the word *mathematics* alone as we feel that the use of the word *numeracy* merely adds a layer of confusion and fuzziness to the issues being discussed. However, since some of our respondents have used the word *numeracy* and the word is used in some official reports, it is difficult to avoid its use altogether.

Statistics

Statistics is the science of the collection, organization, analysis and interpretation of data. It deals with all aspects of this, including the planning of data collection in terms of the design of surveys and experiments. Statistics is often used to make inferences about systems based on incomplete information of the systems in the first place (for example, on information obtained from a sample of the whole).

Functional Mathematics

Functional Skills are qualifications developed by the UK Government in order to improve literacy, numeracy and ICT skills. They are available as stand-alone qualifications. Functional mathematics assessments will consist of mathematical problem-solving and decision-making using numbers with tasks that simulate the natural occurrence of mathematics within real-life contexts.

Basic mathematics

This term was originally coined when research (for the Moser report in 1999)[24] revealed the substantial numbers of adults in the UK who lacked capability in literacy and in numeracy. Programmes of basic skills provision aimed to improve the skills of adults. 'Basic skills' was rebranded by the UK Government in 2001 as part of the Skills for Life initiative, and adults and young people over 16 entering courses in FE colleges will often take a diagnostic test on enrolment to see if they would benefit form programmes that address levels of mathematics below GCSE Grade C.

23 For a full review of the research in the area of numeracy and for an indication of the various positions adopted on the relation between numeracy and mathematics, see Coben (2003).

24 The findings in this ACME report indicate a strong disagreement with the Moser report on the level of mathematics required to function at a satisfactory level in the workplace (and, by implication, in society in general).

APPENDIX 5 ACCEPTANCES ONTO HIGHER EDUCATION COURSES IN 2009

The table below shows the total number of acceptances onto HE courses in 2009; and those acceptances who had at least one A-level pass.

HE acceptances for UK domiciled applicants – 2009	Total accepted	Acceptances with A-level	Accepts with A-level / Total accepts
JACS2 Subject Group			
A Medicine and dentistry total	8,254	5,621	68.10%
B Subjects allied to medicine total	45,193	14,712	32.55%
C Biological sciences total	34,316	21,115	61.53%
D Veterinary science, agriculture and related total	5,105	2,092	40.98%
F Physical sciences total	15,803	11,858	75.04%
G Mathematical and computer science total	24,920	12,669	50.84%
H Engineering total	18,313	9,860	53.84%
J Technologies total	2,746	1,117	40.68%
K Architecture, building and planning total	8,634	4,459	51.64%
L Social studies total	32,419	17,092	52.72%
M Law total	18,688	12,437	66.55%
N Business and administrative studies total	45,190	24,175	53.50%
P Mass communications and documentation total	9,973	6,301	63.18%
Q, R, T Linguistics, classics, languages	17,752	14,267	80.37%
V History and philosophical studies total	14,148	11,196	79.13%
W Creative arts and design total	48,684	21,765	44.71%
X Education total	15,901	6,691	42.08%
Y Combined arts total	12,714	8,732	68.68%
Y Combined sciences total	7,029	3,628	51.61%
Y Combined social sciences total	4,525	3,075	67.96%
Y and Z Combinations of subjects from different groups	34,756	18,334	52.75%
	425,063	**231,196**	**54.39%**

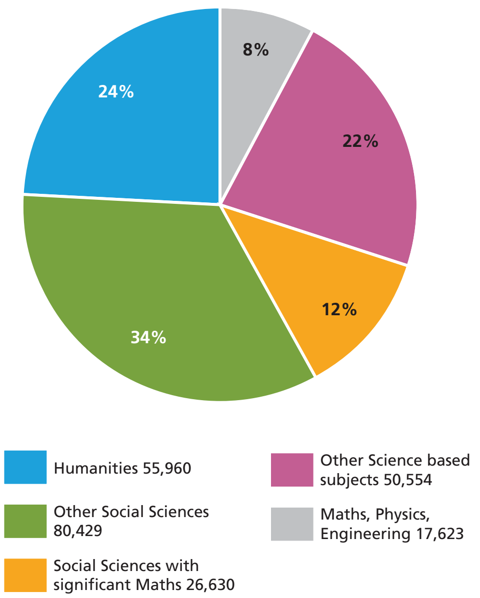
The proportion of those progressing to HE with A-level varies by Subject Group. This variation is even more pronounced when individual subjects are considered, and the table below gives some selected examples to show this variation.

HE acceptances for UK domiciled applicants – 2009	Total accepted	Acceptances with A-level	Accepts with A-level / Total accepts
JACS2 Subject Line			
B7 Nursing	24,636	4,540	18.43%
D1 Pre-clinical veterinary medicine	725	502	69.24%
D3 Animal science	1,939	755	38.94%
G1 Mathematics	5,809	5,068	87.24%
G4 Computer science	11,841	4,453	37.61%
L1 Economics	5,029	4,320	85.90%
L5 Social work	10,554	1,700	16.11%
X1 Training teachers	7,916	3,839	48.50%

Those subject lines where there are low proportions of students coming from the A-level route reflect the significant number of older entrants supported by their employers joining HE courses (as in social work and nursing) as well as the significant numbers of those taking Level 3 vocational qualifications (as in computer science).

Chart 3 of this report (Section 4.2) shows the proportion of university acceptances for various subject groups. Below is the equivalent Chart for that group of individuals who enter HE with A-levels. As can be seen, the proportions in the two charts are very similar.

University acceptances with A-level, 2009 (231,196 individuals)



For further information about the Advisory Committee on Mathematics Education:

ACME
The Royal Society
6–9 Carlton House Terrace
London SW1Y 5AG
Tel +44 (0)20 7451 2574
Fax +44 (0)20 7451 2693
Email acme@royalsociety.org
Web acme-uk.org

The Royal Society

The Royal Society is a Fellowship of 1400 outstanding individuals from all areas of science, engineering and medicine, who form a global scientific network of the highest calibre. Since its foundation, the Royal Society has played its part in some of the most significant scientific breakthroughs and discoveries.

Today the Society's mission is to expand the frontiers of knowledge by championing the development and use of science, mathematics, engineering and medicine for the benefit of humanity and the good of the planet. In addition to providing support to around 1300 of the best scientists, engineers and technologists in the UK, the Society is active in all aspects of UK science. This includes providing independent scientific advice to Government and other policy makers, supporting science education and innovation, and representing UK science abroad.

The Joint Mathematical Council of the United Kingdom

The Joint Mathematical Council of the United Kingdom was formed in 1963 to: 'provide co-ordination between the Constituent Societies and generally to promote the advancement of mathematics and the improvement of the teaching of mathematics'. In pursuance of this, the JMC serves as a forum for discussion between societies and for making representations to government and other bodies and responses to their enquiries. It is concerned with all aspects of mathematics at all levels from primary to higher education.

THE ROYAL SOCIETY

JMC
Joint Mathematical Council

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Nuffield Foundation

The Clothworkers' Foundation